SERVO MOTORS AND MOTION CONTROL SYSTEMS
(Mechanical Systems-Lecture 2)
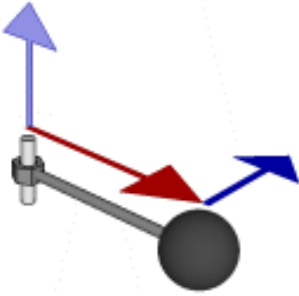
Dr.Caglar UYULAN

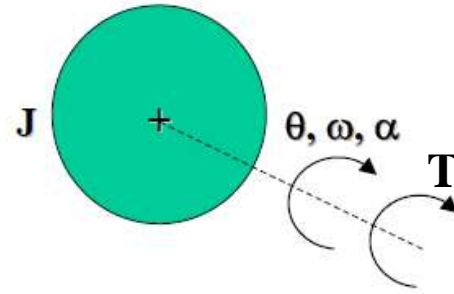| Linear Motion				Rotational Motion		
Quantity	Symbol	Unit		Quantity	Symbol	Unit
position	x, r	m		angular position/angle	θ	rad
velocity	ν	$m \cdot s^{-1}$		angular velocity	ω	s^{-1}
acceleration	a	$m \cdot s^{-2}$		angular acceleration	α	s^{-2}
jerk	j	$m \cdot s^{-3}$		angular jerk	ζ	s^{-3}
mass	m	m		moment of inertia	J	$kg \cdot m^2$
momentum	p	$kg \cdot m \cdot s^{-1}$		angular momentum	L	$kg \cdot m^2 \cdot s^{-1}$
force	F	$kg \cdot m \cdot s^{-2}$		torque	T	$kg \cdot m^2 \cdot s^{-2}$
power	$P=F. \nu$	$kg \cdot m^2 \cdot s^{-3}$		power	$P=T. \omega$	$kg \cdot m^2 \cdot s^{-3}$
energy	$E=m. \nu^2/2$	$kg \cdot m^2 \cdot s^{-2}$		energy	$E=J. \omega^2 / 2$	$kg \cdot m^2 \cdot s^{-2}$

Power & Energy: Laws

Rotational Motion

$$\tau = \mathbf{r} \times \mathbf{F}$$
$$L = \mathbf{r} \times \mathbf{p}$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$T = J \cdot \alpha$$

J $\quad$ + $\quad$ θ, ω, α $\quad$ T

Acc

Vel

Disp
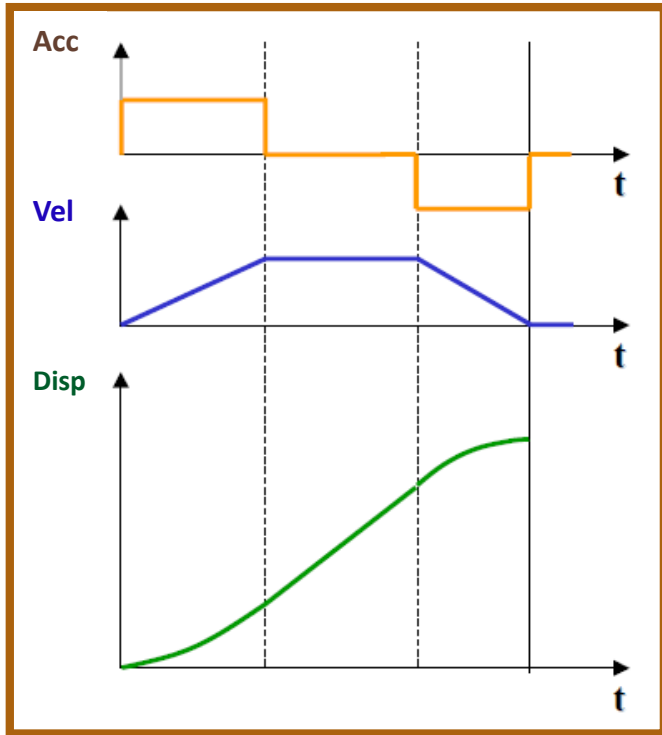
Linear Motion

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt}.$$
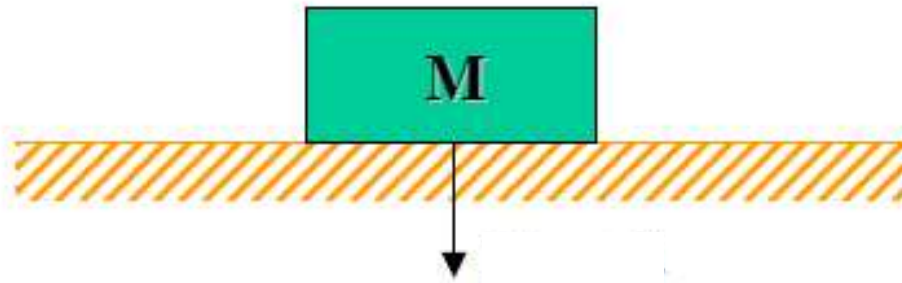
x,v,a

M $\quad$ F
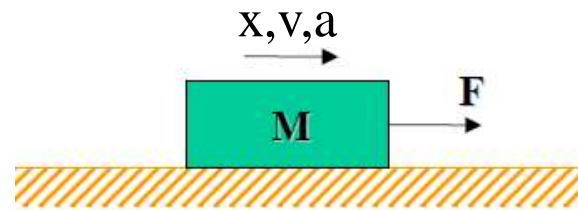
$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$
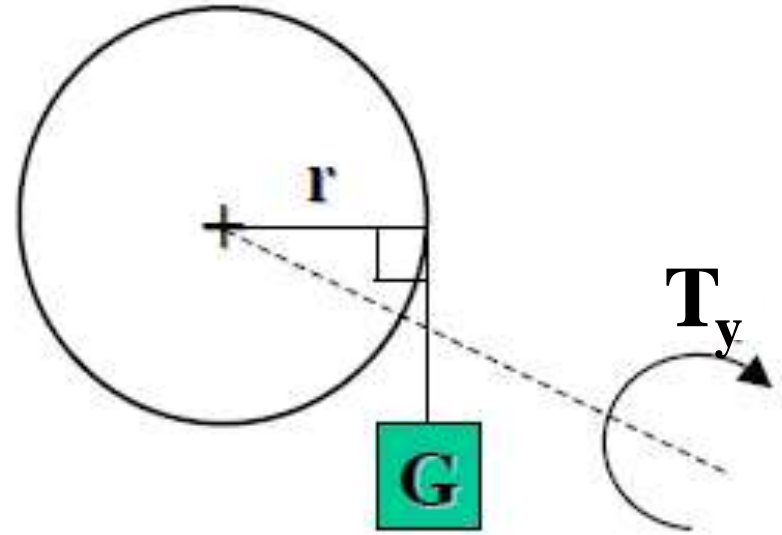
$$F = M \cdot a$$

The causes for the motor load:
1. Gravity
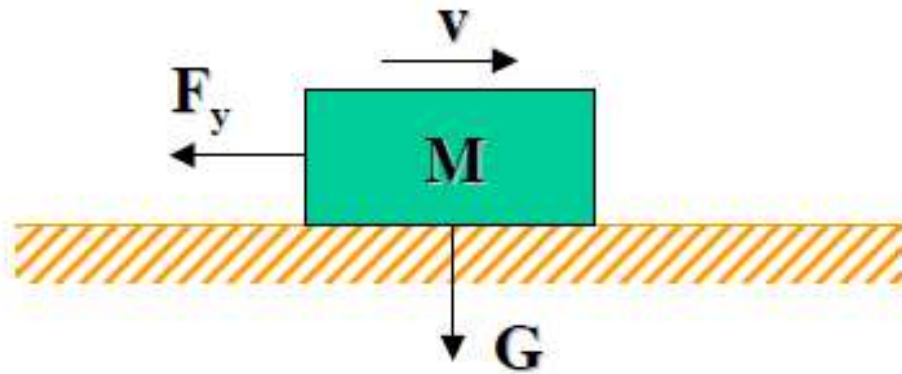
$$F_y = M \cdot g = G$$

$$T_y = M.g.r$$
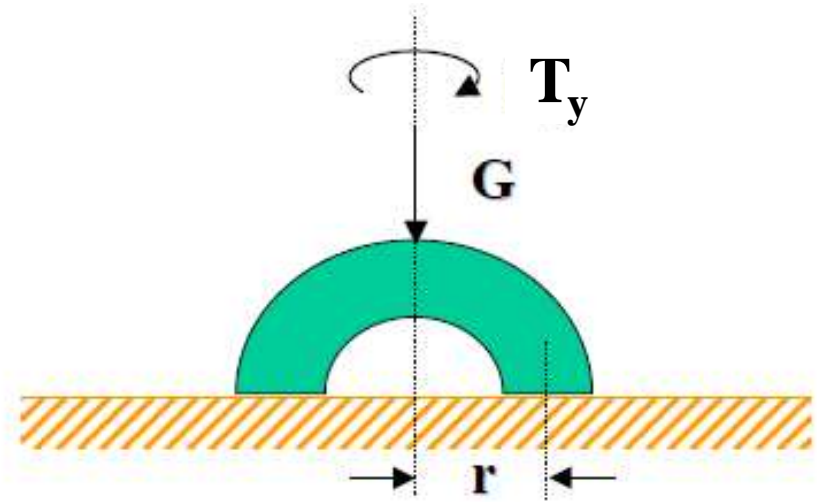
2. Friction (dry)

$$F_y = \mu\, M\, g$$

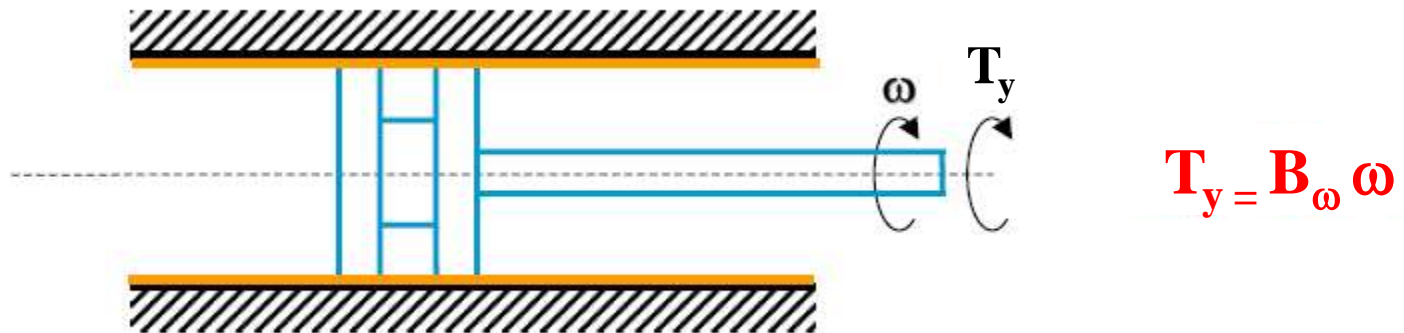$$T_y = \mu\, M\, g\, r$$

The friction coefficient has two different values, one at rest and the other in motion. Sticktion when the load is at rest, μst, b) slip when the load is in motion, μsl , μ <1

Load - Surface	Sticktion μ_{st}	Slip μ_{sl}
Steel – Steel	0.78	0.42
Aluminum – Steel	0.61	0.47
Glass – Glass	0.94	0.40
Tyre – Road	0.90	0.85

3. Friction (viscous)

$$F_{y\,=}\,B_v\,v$$

$$T_{y\,=}\,B_\omega\,\omega$$
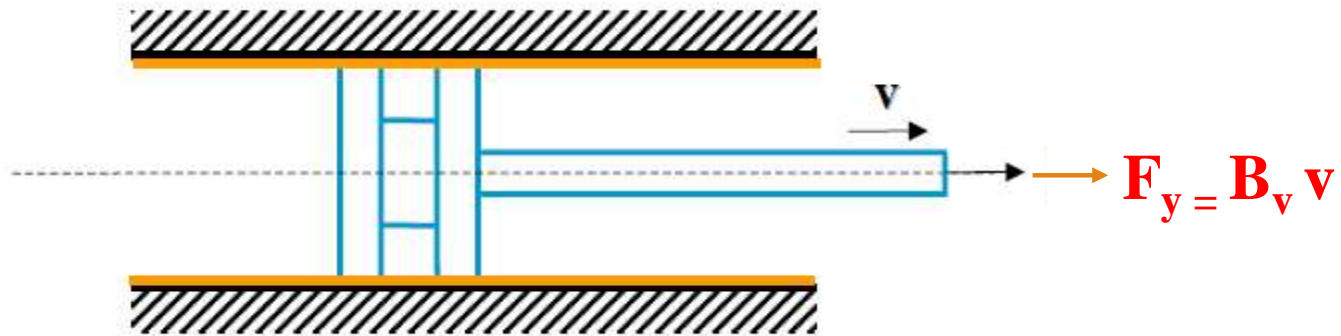
4. Friction (turbulence)

$$T_y = \sigma \cdot \omega^2$$
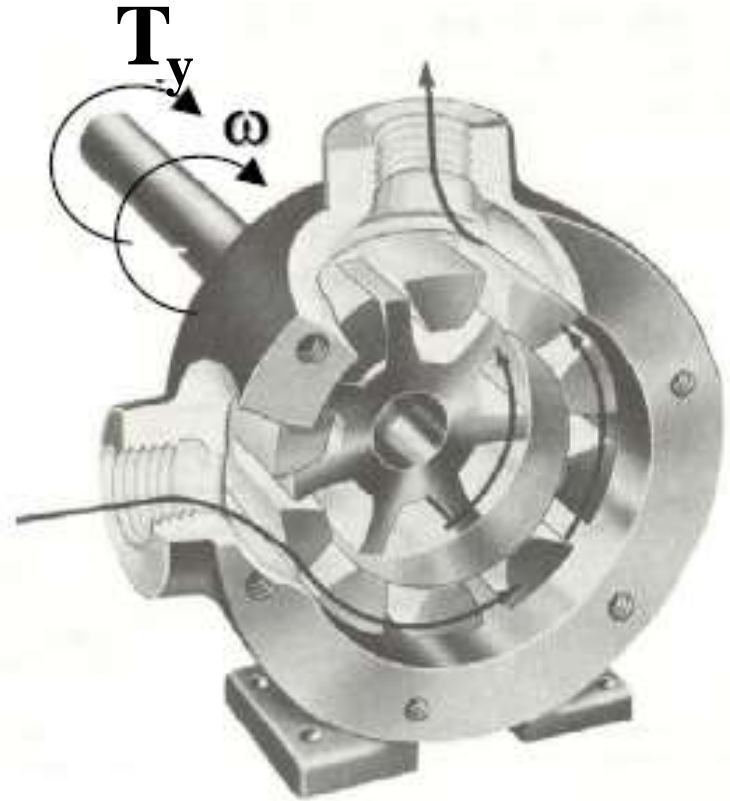
Friction / Loss in fans and pumps

5. Spring compression/tension
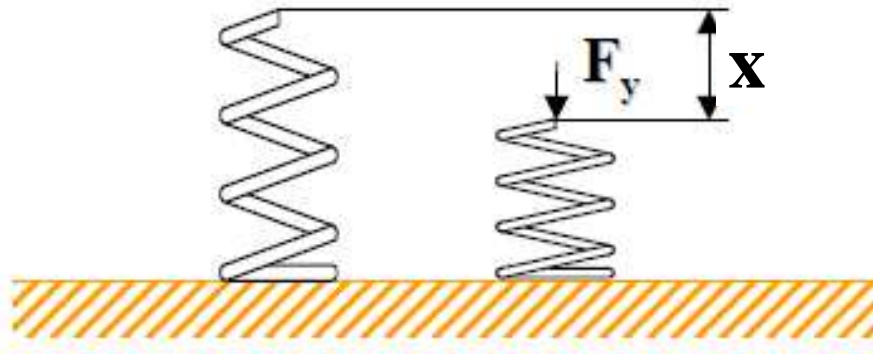
5. Spring torsion

$$F_{y=} K_x \, x$$
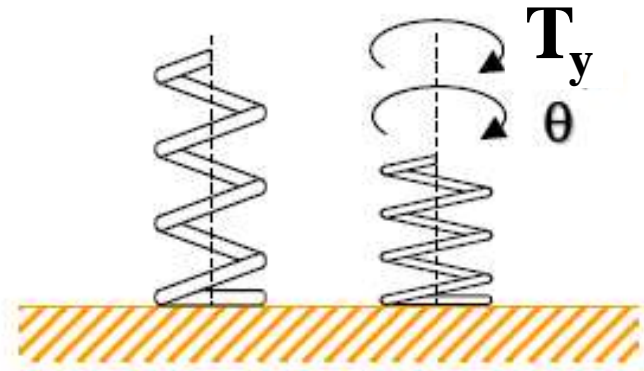
$$T_{y=} K_\theta \, \theta$$

Transmitting Motion

Gearbox (Reducers)
Belt-Pulley
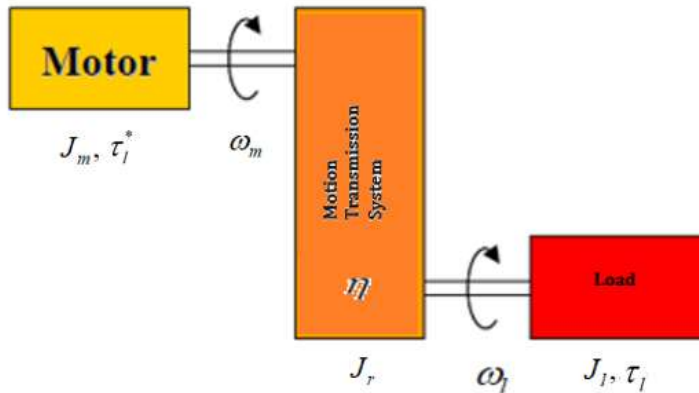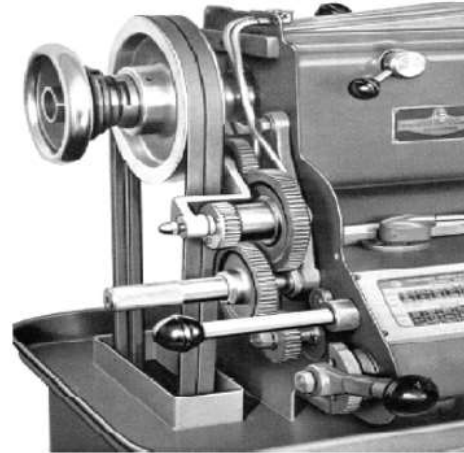
$$\eta = \frac{\omega_l}{\omega_m}$$ ⟹ Conversion ratio

$$\tau_l^* = \tau_l \eta$$ ⟹ Reduced load torque on motor shaft

$$J = J_m + J_r + J_l\left(\eta^2\right)$$

Equivalent inertia in the motor shaft
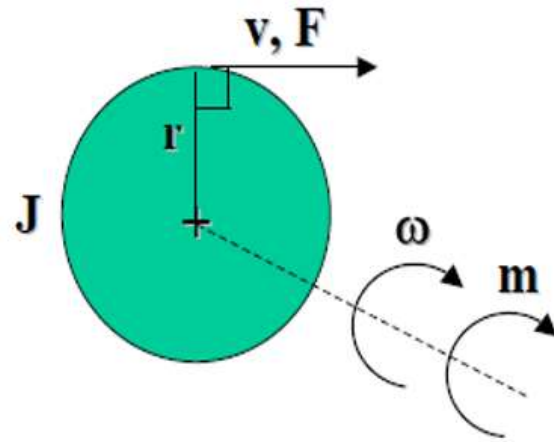
$$v = r \cdot \omega \implies \text{Tangential velocity}$$

$$T = F \cdot r \implies \text{Applied torque}$$

Centrifugal force on a point mass rotating about an axis
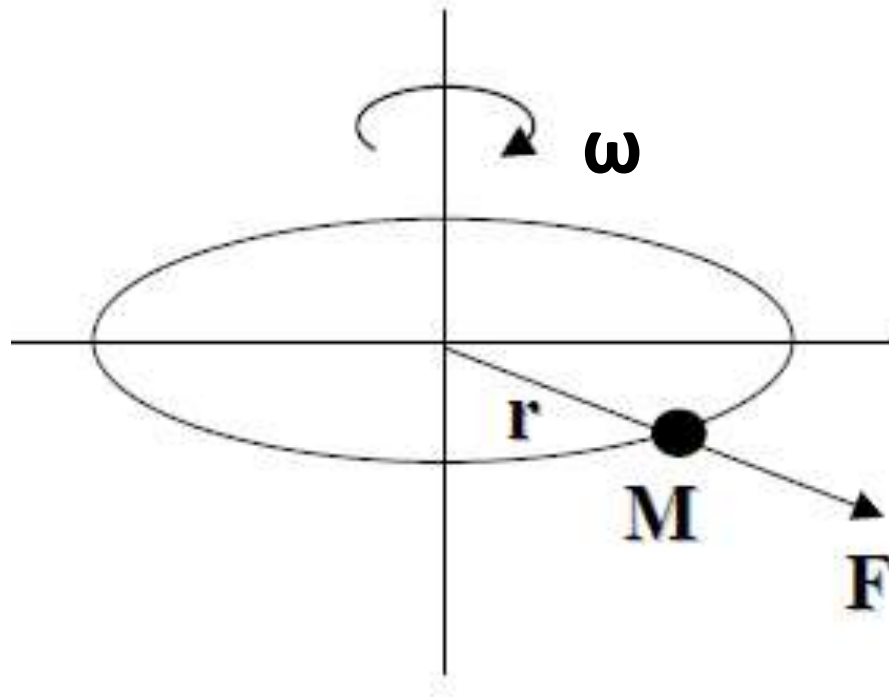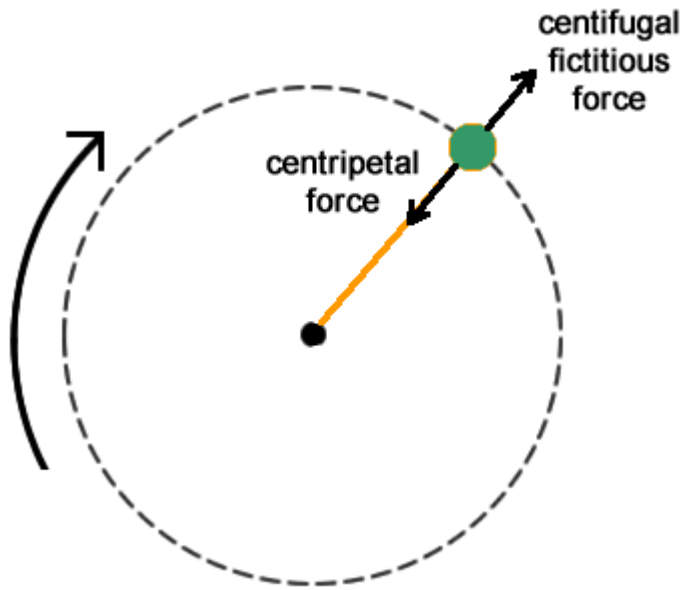
$$F = M \cdot r \cdot \omega^2$$

Mass and Moment of Inertia(MoI)

Mass by definition is the phenomenon of the body's resistance to being accelerated by a force.

The moment of Inertia is the resistance of the mass to rotational force.

MoI is calculated concerning the axis of rotation.

Energy is stored by the motion of Mass and Moment of Inertia.

Moment of Inertia of a point mass rotating about an axis

$$J = \int_0^M r^2 dm$$
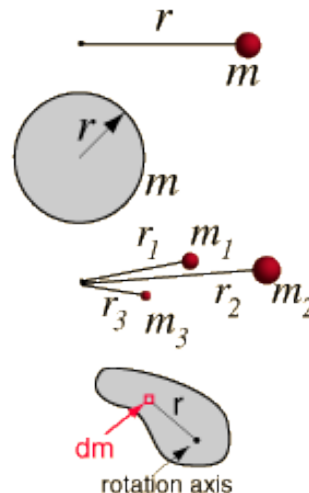
$$I = mr^2$$

For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.

$$I = kmr^2$$

For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius r.

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

Sum of the point mass moments of inertia.

$$I = \int_0^M r^2 dm$$

Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.

dm

rotation axis

• Every object that moves in a circular motion around a certain axis has a moment of inertia.

• The inertia of the object varies according to the axis of rotation.

• In circular motion, a moment of inertia has the function of mass in linear motion.

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\frac{d\vec{\omega}}{dt}\vec{r}$$

$$\vec{T} = \vec{r} \times \vec{F} = \vec{r} \times \left(m\frac{d\vec{\omega}}{dt}\vec{r} \right) = J\frac{d\vec{\omega}}{dt}$$

$$J = m\left| \vec{r} \right|^2$$

$$I_z = \frac{mr^2}{2}$$

$$I_x = I_y = \frac{mr^2}{4}$$

$$I_z = mr^2$$
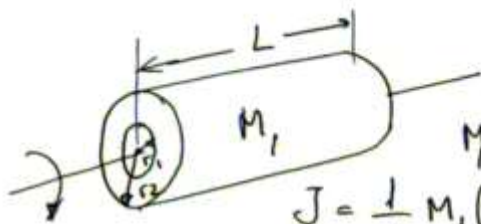
$$I_x = I_y = \frac{mr^2}{2}$$
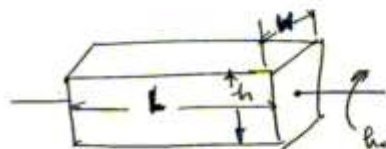
Parallel-Axis Theorem

$$J = J_{cm} + md^2$$

$$I_{end} = \frac{mL^2}{3}$$

$$I_{center} = \frac{mL^2}{12}$$

$$I = mr^2$$
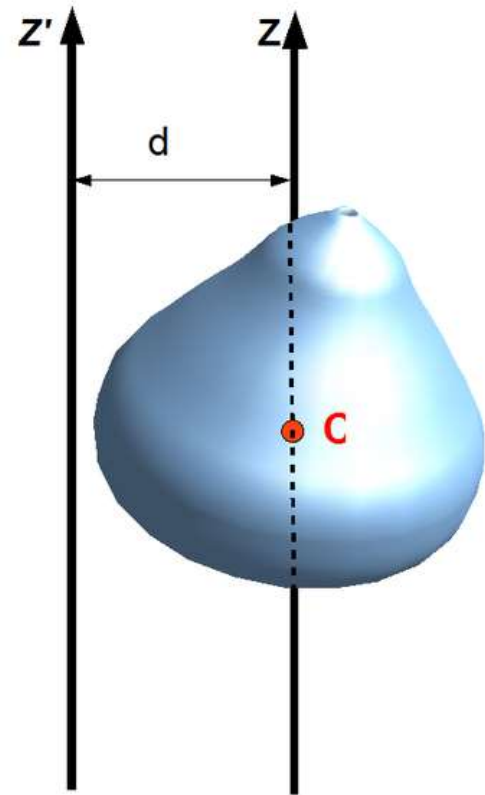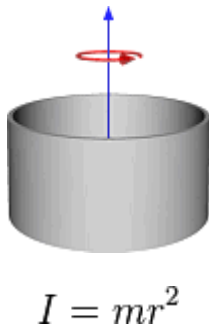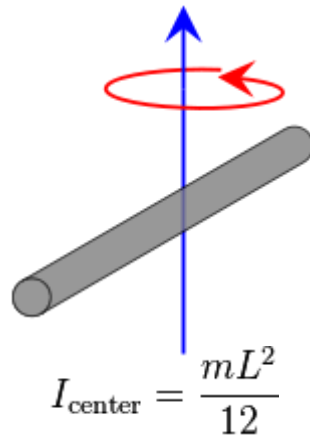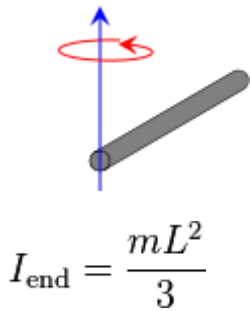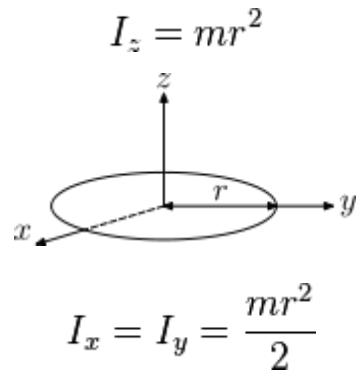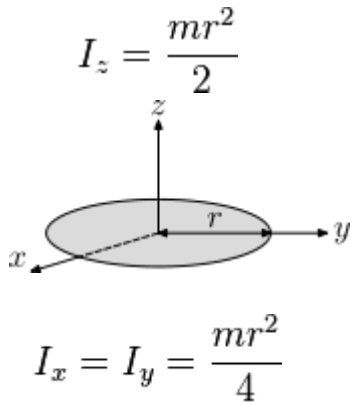
$$M = \rho \pi L (r_2^2 - r_1^2)$$

$$J = \frac{1}{2} M_1 (r_2^2 + r_1^2)$$

$$= \frac{\rho L \pi}{2} (r_2^4 - r_1^4)$$

$$J = \frac{M}{12} (h^2 + w^2)$$

$$V = L \cdot h \cdot W$$

$$M = V \cdot \rho$$

$$\rho \left[\frac{kg}{m^3} \right]$$
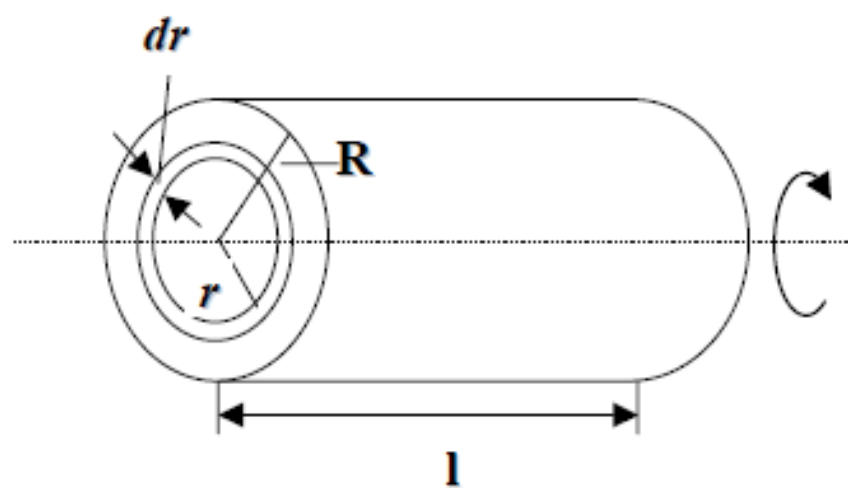
$$dM = 2\pi \cdot r \cdot l \cdot \gamma \cdot dr$$

$$v = r \cdot \omega$$

$$dF = dM \cdot dv/dt = dM \cdot r \cdot d\omega/dt$$

$$dm = r \cdot dF = dM \cdot r^2 \cdot d\omega/dt$$

$$m = d\omega/dt \cdot \int r^2 \cdot dM$$

$$m = \pi/2 \cdot \gamma \cdot l \cdot R^4 \cdot d\omega/dt = J \cdot d\omega/dt$$

$$J = \pi/2 \cdot \gamma \cdot l \cdot R^4 = M \cdot R^2 / 2$$

Hollow

L, $D_o = 2r_o$, $D_i = 2r_i$

$$J = \frac{M(D_o^2 + D_i^2)}{8} = \frac{G(r_o^2 + r_i^2)}{2g} = \frac{\pi L \rho (r_o^4 - r_i^4)}{2g}$$

$$\text{Volume} = \frac{\pi}{4}(D_o^2 - D_i^2) L$$

Solid

L, $D = 2r$
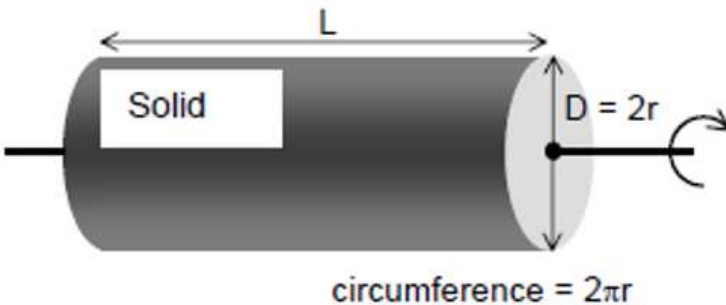
$$J = \frac{M D^2}{8} = \frac{G r^2}{2g} = \frac{\pi L \rho r^4}{2g}$$

$$\text{Volume} = \pi r^2 L$$

circumference = $2\pi r$

Solid

w, h, L

$$J = \frac{G}{12g}(h^2 + w^2)$$

$$\text{Volume} = L\,h\,w$$

•**Degrees of freedom (DOF):** It is the number of masses or inertia that can move relative to each other in a dynamic system.

•In this case, in-motion translation systems or rotational systems.

$$J\ddot{\theta} = \sum T_y$$

$$M\ddot{x} = \sum F_y$$

For example, in a 2 degree of freedom system consisting of 2 masses
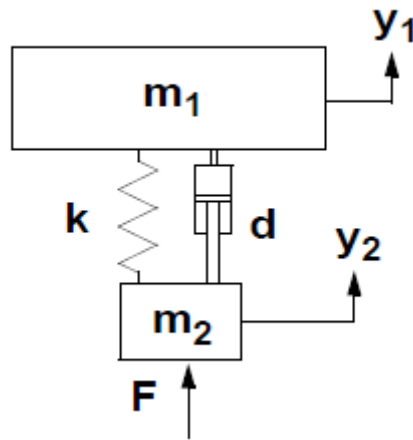
$$m_1\ddot{y}_1 = \sum F_{1y} = k(y_2 - y_1) + d(\dot{y}_2 - \dot{y}_1)$$
$$m_2\ddot{y}_2 = \sum F_{2y} = F - k(y_2 - y_1) - d(\dot{y}_2 - \dot{y}_1)$$

Modeling and dynamic analysis of mechanical systems:

- **Writing differential equations for the subsystems in the system**

- **Linearization**

- **Representation**

- In State Space; with equations in matrix form or with Input-Output transfer functions

$$\frac{dx}{dt} = f(x, u) \qquad y = h(x)$$

Time domain

$$y(t) = \sum A_i e^{-\lambda_i t} \cos(\omega_i t + \phi_i)$$

$$G(s) = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

Im

+

Re

+

Complex plane

Frequency domain
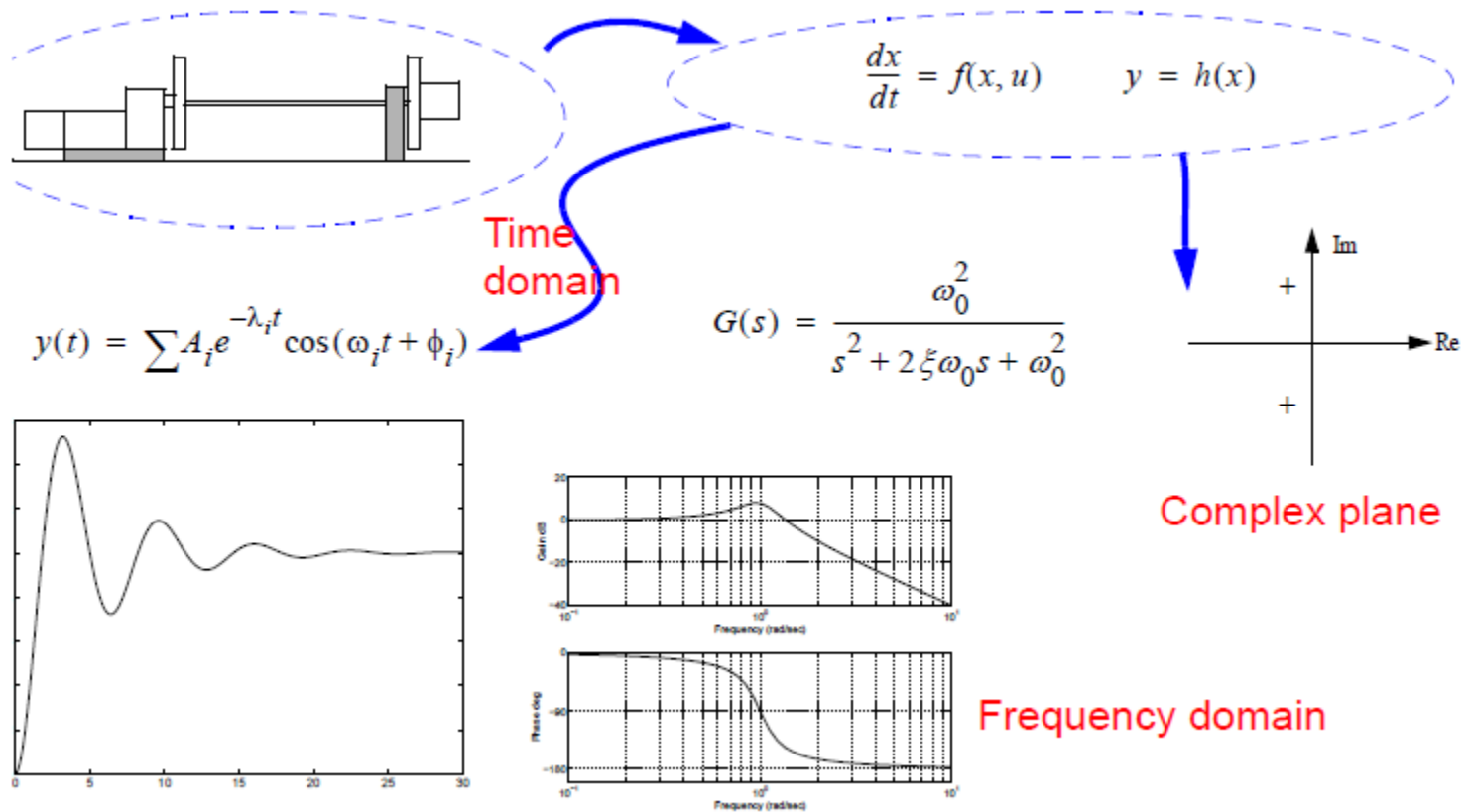
Example: Mass-Spring-Damper (viscous friction)

$$F_s = k.x$$

$$F_i = m.a$$

$$F_b = B.v$$
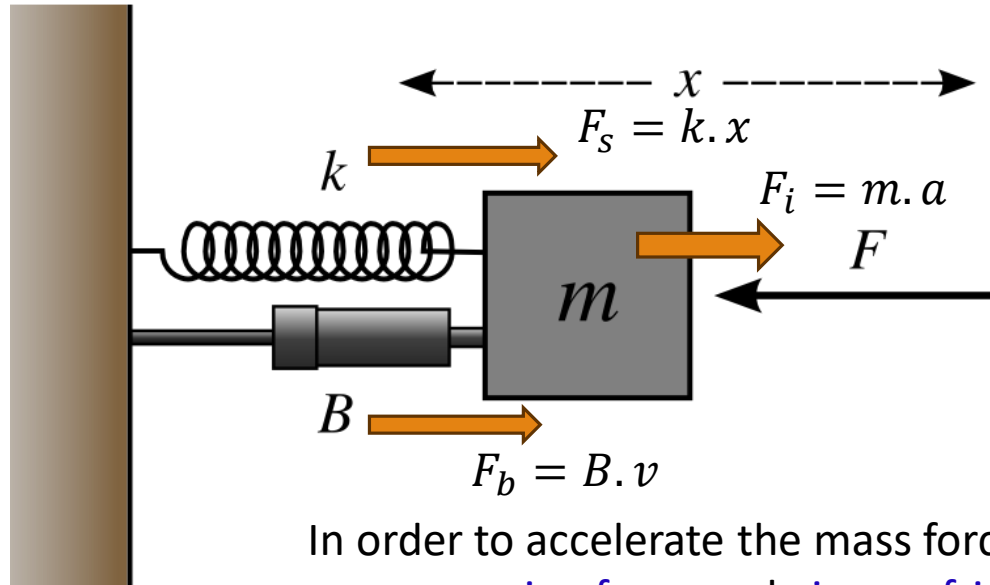
In order to accelerate the mass force must overcome
spring force and viscous friction force

$$F - F_y = m.a$$

$$F_y = k.x + B.v$$

input

Governing equation

$$\text{F} = k.x + B.v + m.a$$

output

$$v = \frac{dx}{dt} = \dot{x}$$

$$a = \frac{d^2x}{dt^2} = \ddot{x}$$

$$F(t) = kx(t) + B\dot{x}(t) + m\ddot{x}(t)$$

$$f(t) = u(t) \ [\text{N}]$$

$$X(s) = \frac{1}{s(ms^2 + Bs + k)} \quad \text{Transfer Function}$$

$$x_\infty = \lim_{s \to 0} s\, X(s) = \lim_{s \to 0} s\, \frac{1}{s(ms^2 + Bs + k)} = \frac{1}{k}$$

$x(t)$

Under damped

Critically damped

$\dfrac{1}{k}$

Over damped
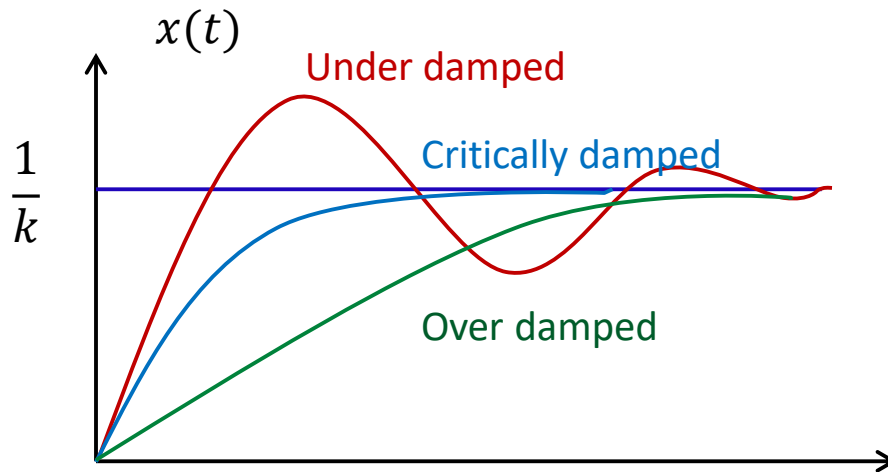
Unstable ?

Continuous time

differential equations

$$\dot{y} = -ay + bu$$

state space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Transfer func. (Laplace)

$$y(s) = \frac{b}{s+a} u(s)$$

Discrete time

difference equations

$$y[n] = ay[n-1] + bu[n]$$

state space model

$$x[n+1] = \Phi x[x] + \Gamma u[n]$$
$$y[n] = Cx[n] + Du[n]$$

Transfer func. (z-transform)

$$y(z) = \frac{b_1 z + b_0}{z+a}$$

$$X_{\infty} = \lim_{s \to 0} s \cdot X(s) = \frac{1}{K}$$
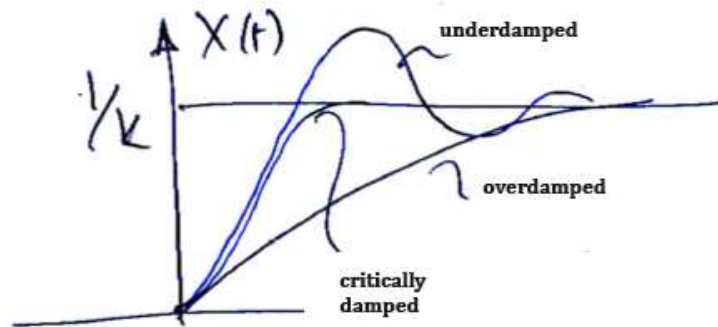
$$X(s) = \frac{1}{s(Ms^2 + Bs + K)}$$

$$f(t) = u(t) \quad [N]$$

Continuous time models

- Differential equations - this is the starting point

$$\frac{dy}{dt} = f(y, t) + g(u)$$

- State space formulation - convenient for larger systems

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- Transfer functions, input/output models, for SISO systems

$$G(s) = C(SI - A)^{-1}B \quad \rightarrow \quad G(s) = \frac{b_m s^m + \ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}$$

- Block diagrams - good for physical insight

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

$$\frac{dx}{dt} = V$$

$$\frac{dv}{dt} = -\frac{B}{M}V - \frac{K}{M}x + \frac{1}{M}F$$

$$\frac{d}{dt}\underbrace{\begin{bmatrix} X \\ V \end{bmatrix}}_{\dot{X}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} X \\ V \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}}_{B}\underbrace{F}_{u}$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C}\underbrace{\begin{bmatrix} X \\ V \end{bmatrix}}_{X} + \underset{0}{\underset{\uparrow}{D}} \cdot u$$
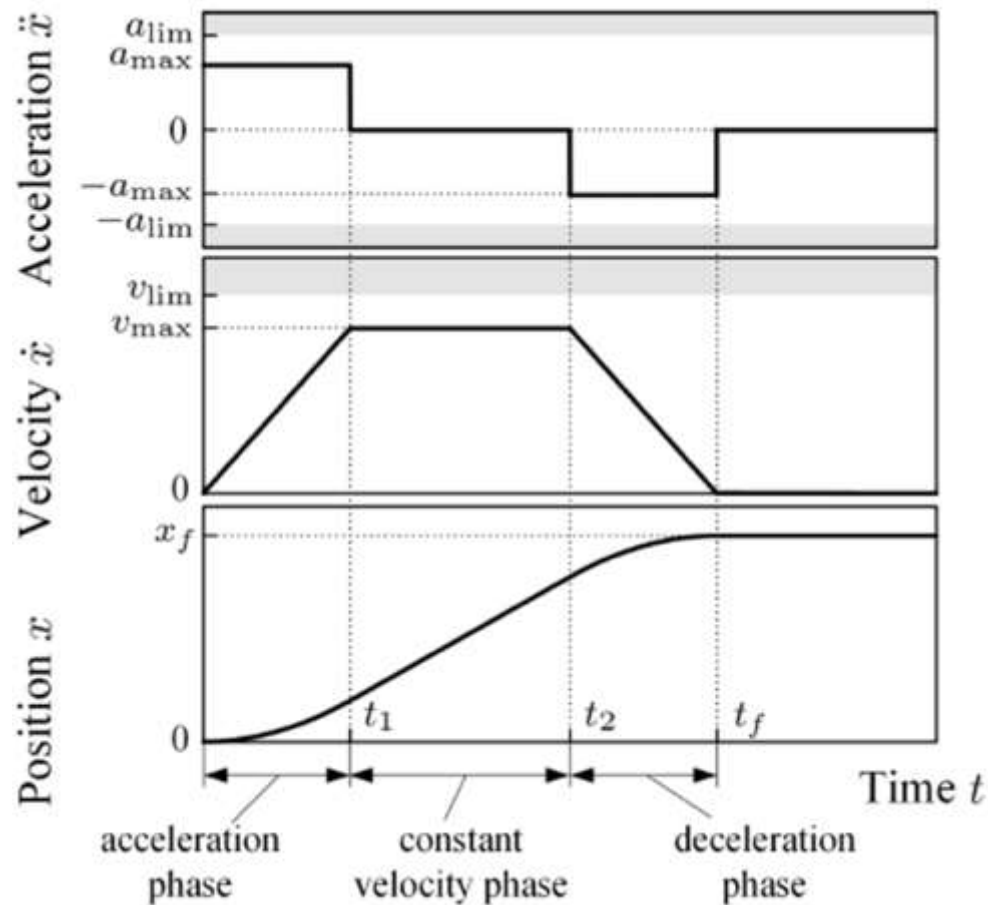
$$\ddot{x}(t) = \begin{cases} a_{\max}, & 0 \le t < t_1 \\ 0, & t_1 \le t < t_2 \\ -a_{\max}, & t_2 \le t \le t_f \end{cases}$$

$$\dot{x}(t) = \begin{cases} a_{\max}t, & 0 \le t < t_1 \\ v_{\max}, & t_1 \le t < t_2 \\ a_{\max}(t - t_f), & t_2 \le t \le t_f \end{cases}$$

$$x(t) = \begin{cases} \frac{1}{2}a_{\max}t^2, & 0 \le t < t_1 \\ \frac{1}{2}v_{\max}(2t - t_1), & t_1 \le t < t_2 \\ x_f - \frac{1}{2}a_{\max}(t_f - t)^2, & t_2 \le t \le t_f \end{cases}$$

$$v_{\max} = \frac{x_f}{t_2}, \quad a_{\max} = \frac{x_f}{t_1 t_2}.$$

$$f_r = 0 \qquad R \begin{array}{c} T \\ \omega \\ \theta \end{array} \qquad r > 0$$

$$J_t = 0$$

$$M_2 \square$$

$$M_1 \downarrow g$$

$$M_h = 0$$
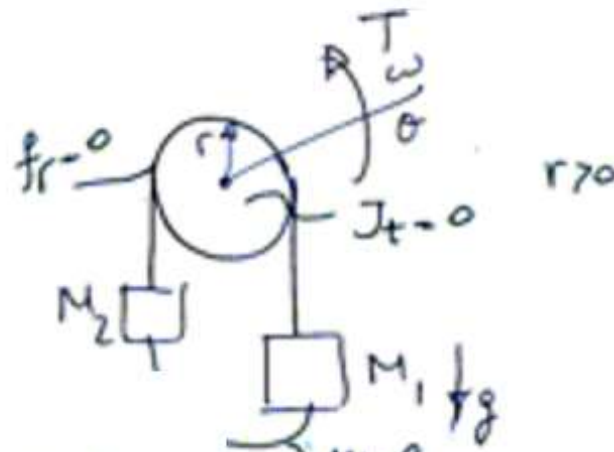
$$T = J\frac{d\omega}{dt} + T_y \qquad T_y = (M_1 - M_2)g \cdot r$$

$$J = (M_1 + M_2) \cdot r^2$$

$$T = \underbrace{(M_1 + M_2)r^2\frac{d\omega}{dt}}_{} + \underbrace{(M_1 - M_2)gr}_{}$$

Load inertia
effect

Gravity effect

Coupling (Transmission Mechanisms)

- In converting rotational motion into translational motion (or vice versa),

- Rotation and the rotation axis are orthogonal,

- Changing the torque or the speed of the motor,

- They are generally used to move the load of the motor with higher torque and lower speed.

Transmission Element Types

Belt-Pulley Systems

Gear Mechanisms
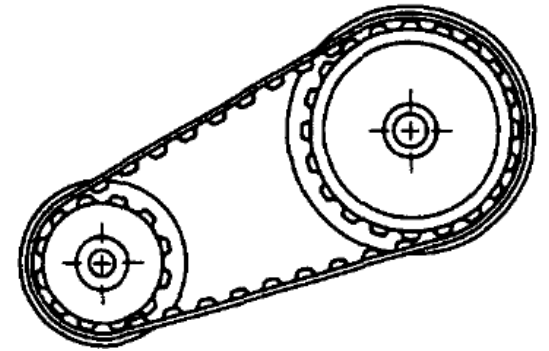
(Courtesy Boston Gear Works)
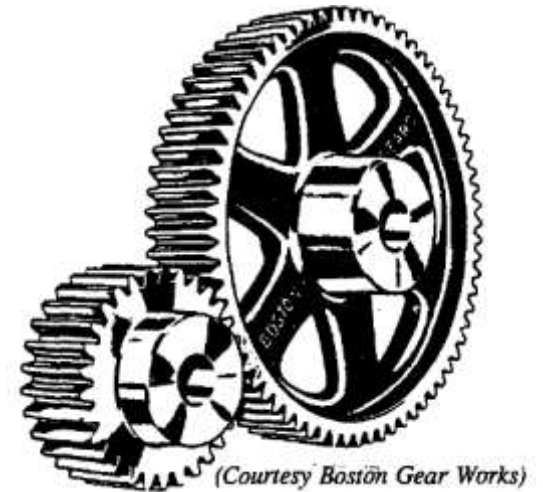
Gears

External vs internal gears

Spur

Helical
  ◦ Skew gears

Double helical

Bevel

Spiral bevels

Hypoid

Crown

Worm

Non-circular

Rack and pinion

Epicyclic

Sun and planet

Harmonic drive

Cage gear

Magnetic gear
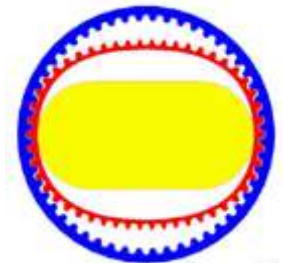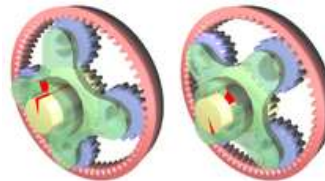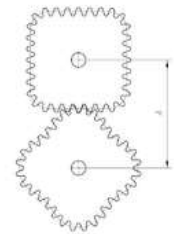
Gears: Fundamentals and Types

Spur Gears

- **Functionality**: Spur gears transmit power through teeth pushing against each other. Despite seeming like they would cause jerky motion, properly shaped teeth roll on each other, ensuring smooth power transfer.

- **Velocity and Torque Alteration**: By meshing gears of different diameters, varying rotational velocities and torque are achieved. The theoretical pitch circle diameter, smaller than the gear's overall diameter due to tooth overlap, is crucial in this regard.

Worm Gears

- **Design and Efficiency**: Comprising a worm and a pinion, worm gears provide high gear ratios with little or no backlash but tend to be inefficient (50-80% efficiency range) due to sliding friction.

- **Lockup Property**: Higher-ratio worm gears have a one-way power flow feature, making them useful in applications like guitar string tuning, where reverse motion is effectively prevented.

Harmonic Drive

- **Unique Characteristics**: Harmonic drives offer high efficiency, no backlash, and high gear ratios. They consist of a rigid circular spline and a flex spline with slightly fewer teeth, meshed via a rotating wave generator.

- **Operation**: The input shaft connects to the wave generator, creating a mesh between the two splines and enabling efficient power transmission.

Clutches and Brakes

Clutches

Purpose: Clutches connect or disconnect rotating shafts while in motion. They are essential in various applications from small devices to industrial machinery.

Mechanism: Typically involving two opposing friction disks, clutches use friction to transmit torque, allowing for controlled engagement and disengagement of the motor and load.

Brakes

Function: Brakes slow down or stop rotating shafts. They resemble clutches in design but have one immovable disk, effectively stopping the shaft when engaged.

Actuation: Brakes can be actuated electromagnetically, pneumatically/hydraulically, or manually.

Belts

Advantages: Belts, usually made of rubber, offer low-cost, quiet operation, and low maintenance. They absorb shock and can accommodate nonparallel shafts.

Transmission Mechanics: Belts allow for power transmission and "gearing" changes through pulley diameter variations. Power is primarily carried by the lower belt segment, and the pulley diameter ratio dictates the change in velocity and torque.
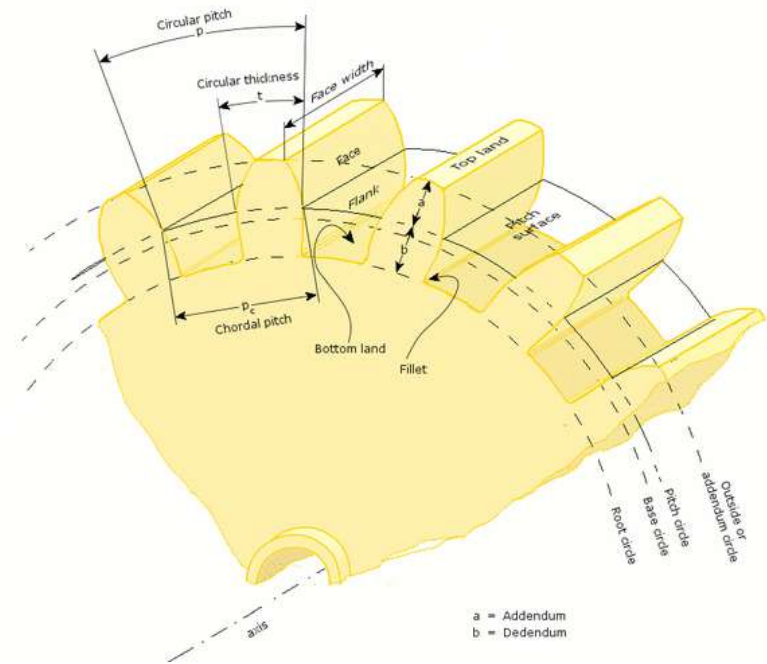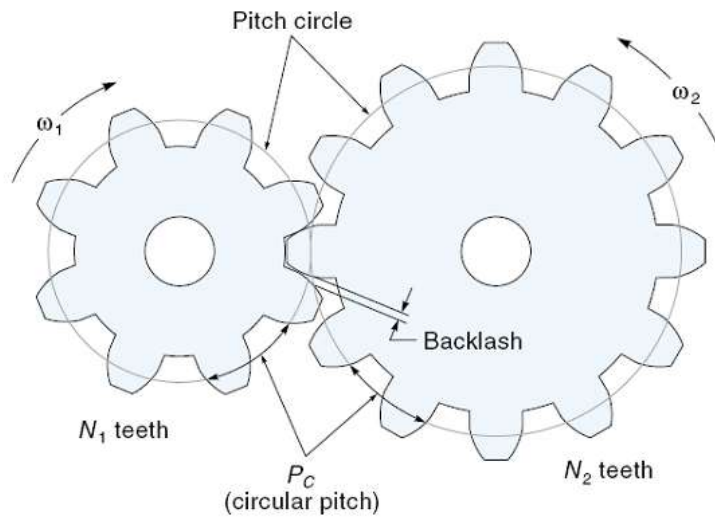
General nomenclature

Rotational to Linear Motion

Rack and Pinion

Ball Screw and Bearing
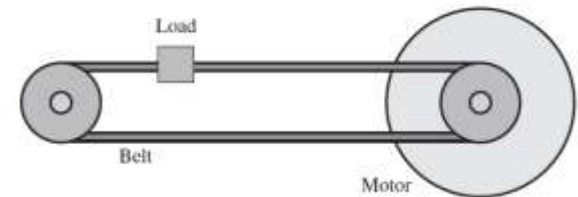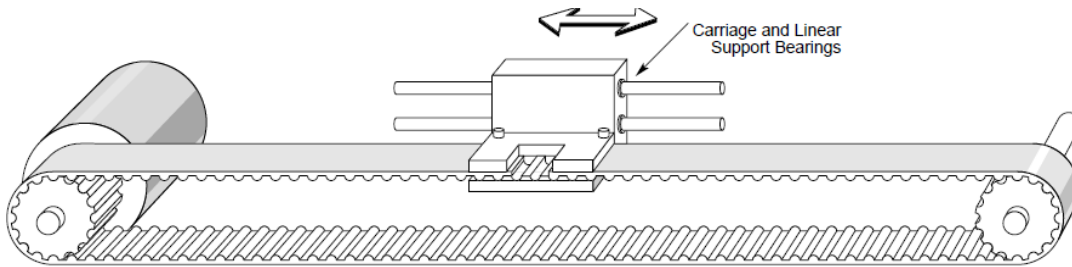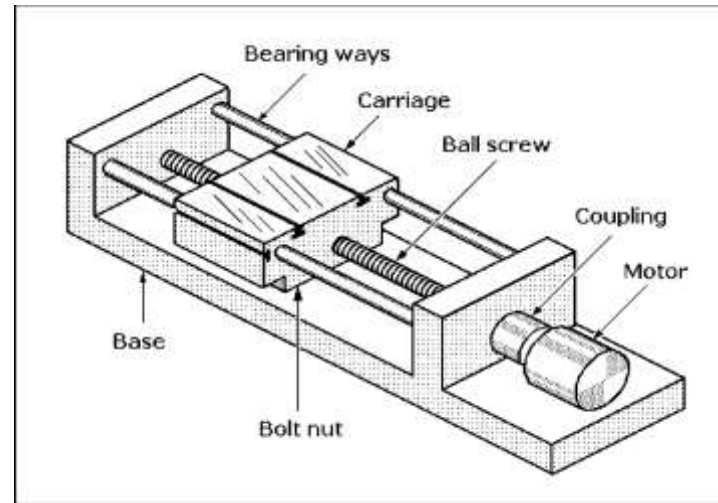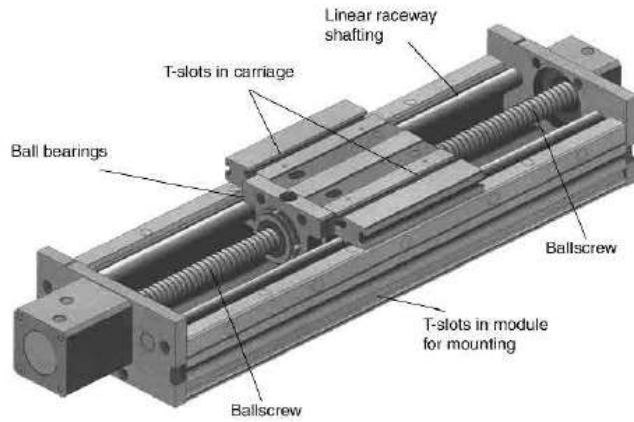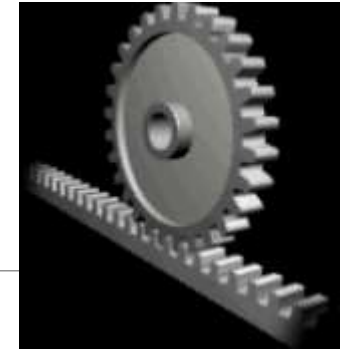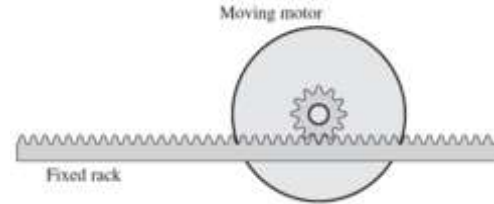
Bevel Gear:

Used to change the rotation angle.

Spur Gear:

It is frequently used due to its simplicity and high efficiency.

Its disadvantage is that it is not resistant to large loads.

Helical (Spiral) Gear: Generally similar to spur gear. Rotates the rotation axis by 90 degrees.

High speeds

Large power transmission

Lower noise

The disadvantage of helical gears is a resultant thrust along the axis of the gear

A greater degree of sliding friction between the meshing teeth is often addressed with additives in the lubricant.
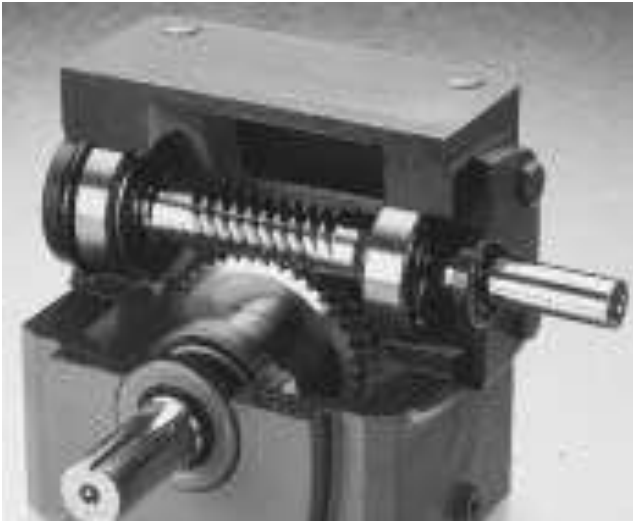
Worm Gear:

Can reach high gear rotation rates.

Another advantage is that this gear cannot be driven backward. This means the motor can turn the main gear. Therefore, gravity or reverse load effects cannot cause any movement.

In this case, there is no need to consume energy to create the holding moment in weight-lifting robots.

Its efficiency is low.

Worm

Worm gear (pinion)
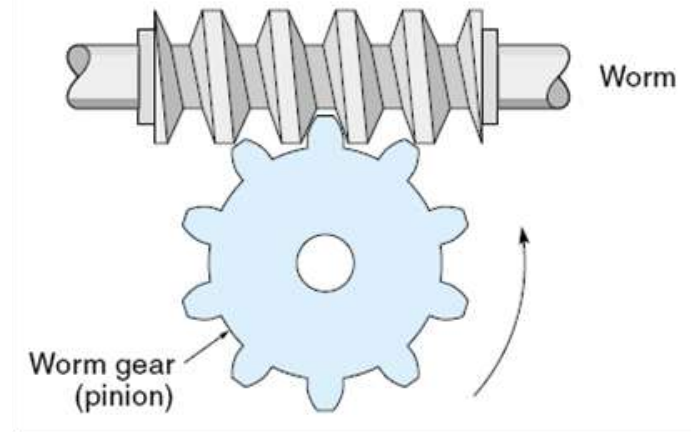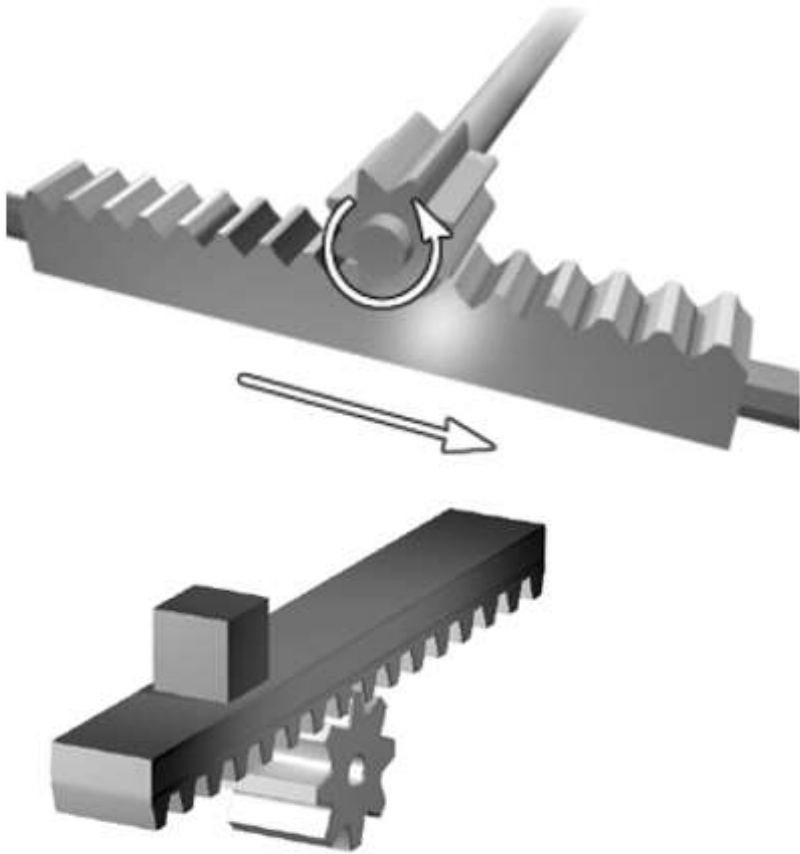
Rack and pinion:

This gear is used in the left and right rotation movements of the vehicle.

It is ideal for converting rotational motion into translational motion.
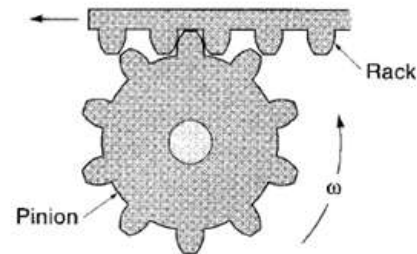
(a) Rack and pinion
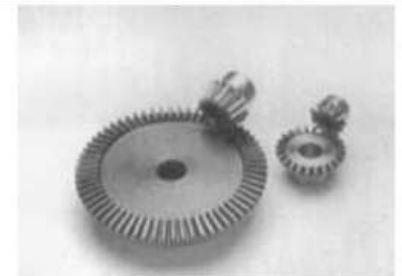
(b) Straight bevel gears

Planetary Gear

Planets

Planet carrier with output shaft

Annulus with internal gear teeth

Input shaft

Central pinion (sun wheel)

SUN GEAR
PLANET PINIONS
SUN GEAR
PLANET CARRIER
RING OR INTERNAL GEAR
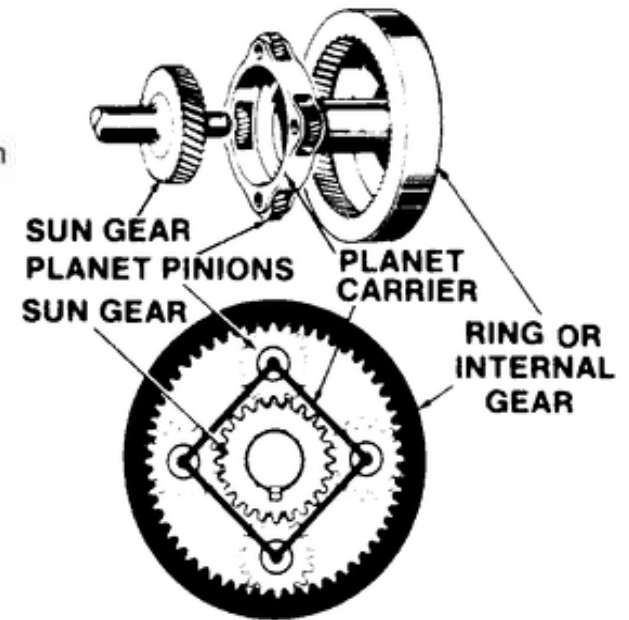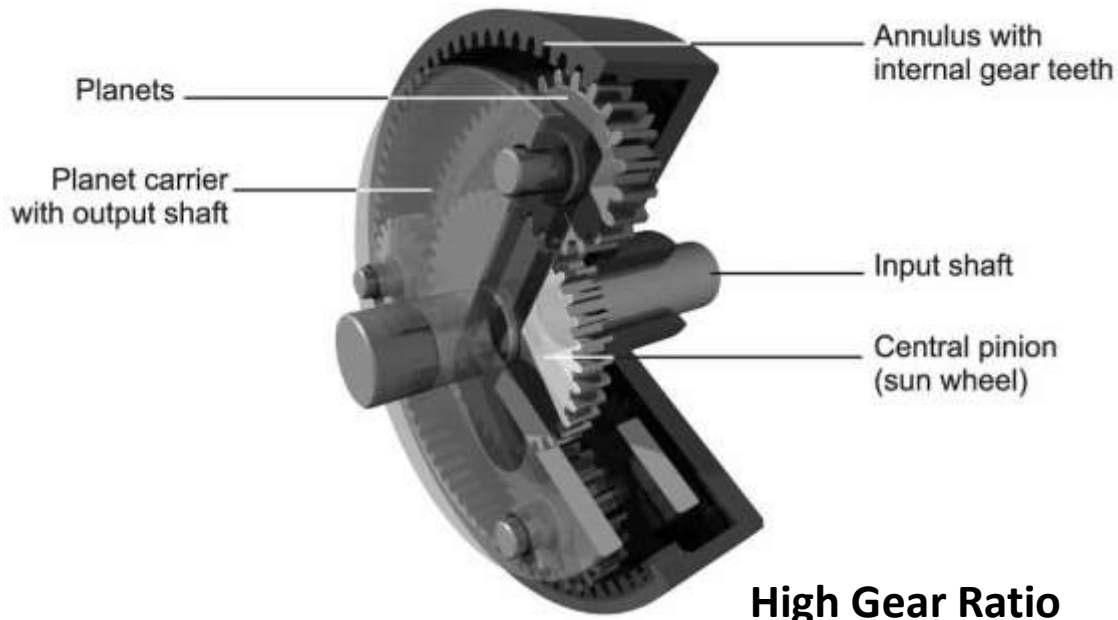
High Gear Ratio

Flexspline
An elliptical, nonrigid, external gear

Circular Spline
A round, rigid, internal gear

Wave Generator
An elliptical ball bearing assembly
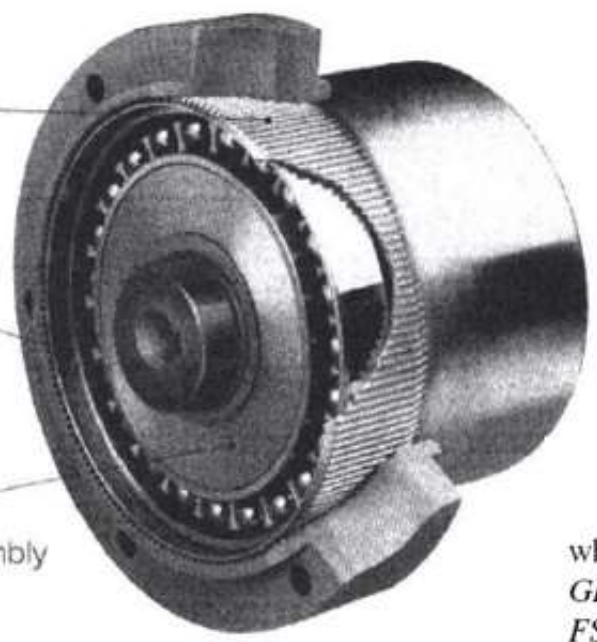
$$GR = \frac{FS}{CS - FS}$$

where:
GR = gear ratio
FS = number of teeth on the flexspline
CS = number of teeth on the circular spline

0°

90°

180°
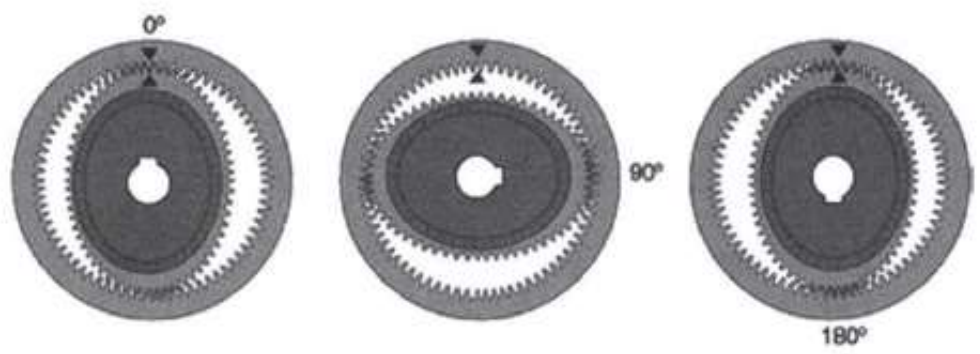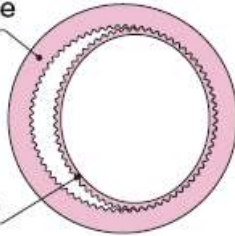
Circular Spline

Flexspline

This condition is called "dedoidal".
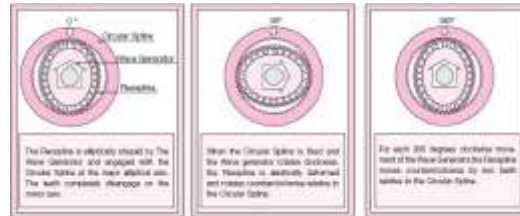
Flexspline
An elliptical, nonrigid, external gear

Circular Spline
A round, rigid, internal gear

Wave Generator
An elliptical ball bearing assembly
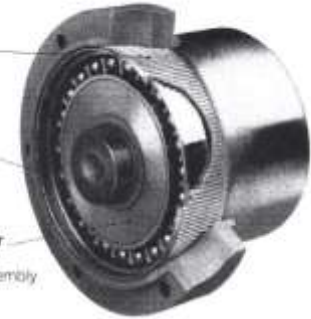
Harmonic drive
High slew rate
Zero Backlash effect

$$N_h = \frac{\text{number of teeth on outer rigid cup}}{\text{difference in number of teeth on splines}}$$

Helical planetary gear

Rigid sun gear

Bevel gear

Bearings

Sealed unit

Bearings

Balanced pinion
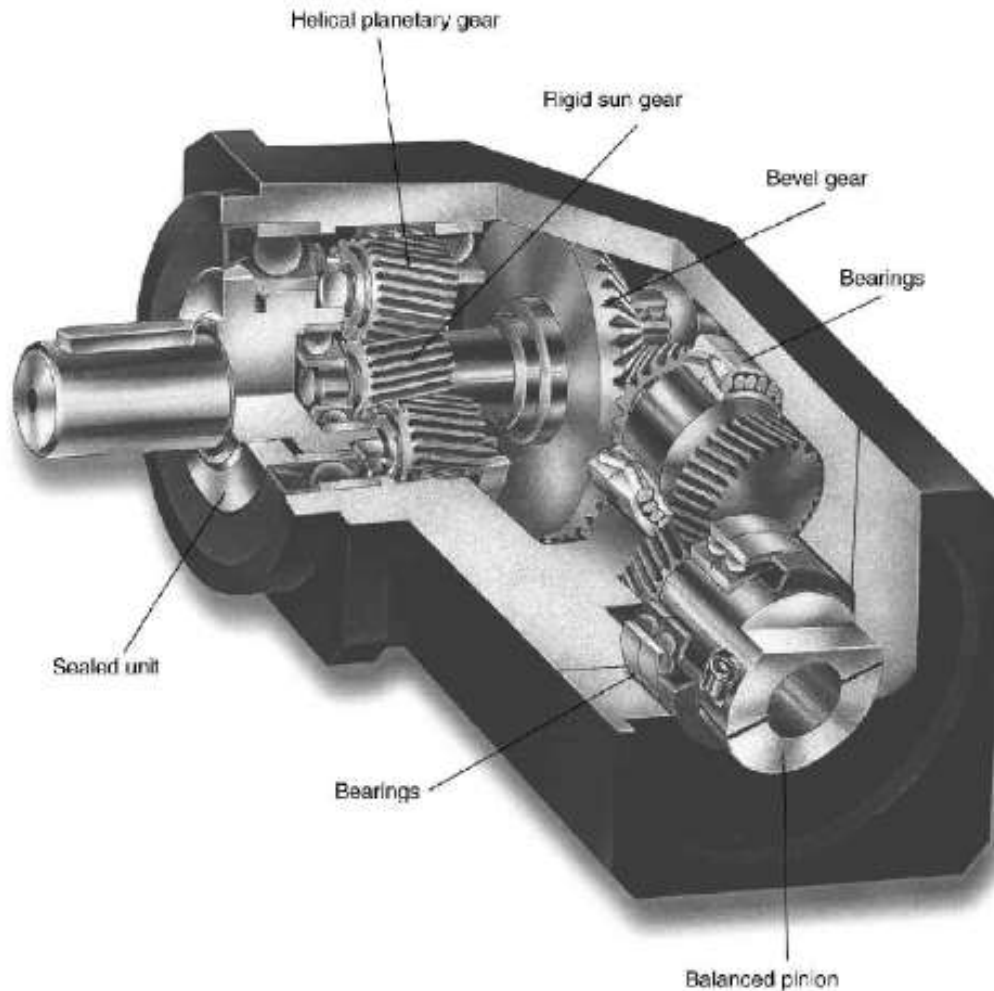
An example of a hybrid mechanism with multiple gear groups

Clutch

Load shaft

Motor

Engaged

Motor

Disengaged

a

b

100 rpm

1200 rpm

A

B

C

D

80 teeth

60 teeth

20 teeth

Bearing ways

Carriage

Ball screw

Coupling

Motor

Base

Bolt nut
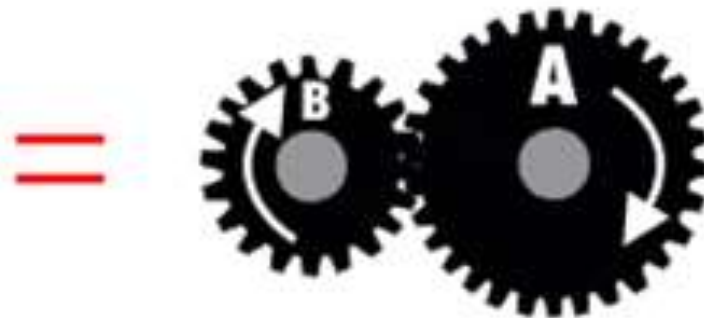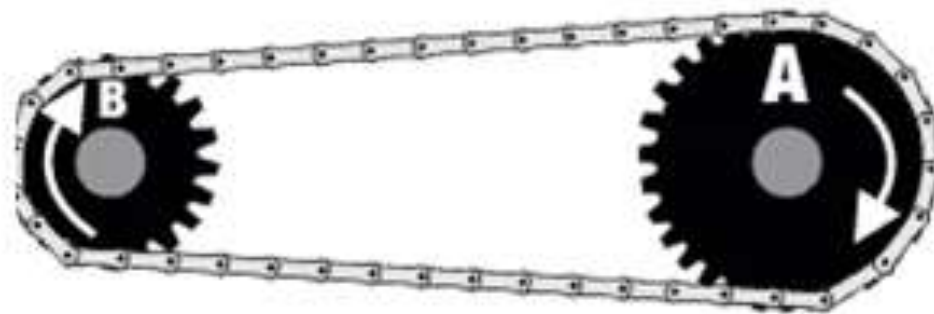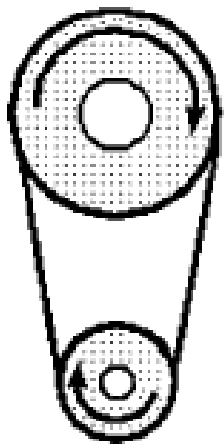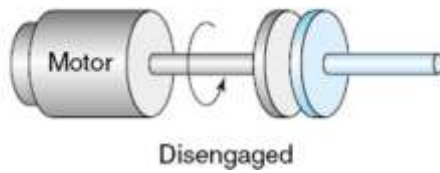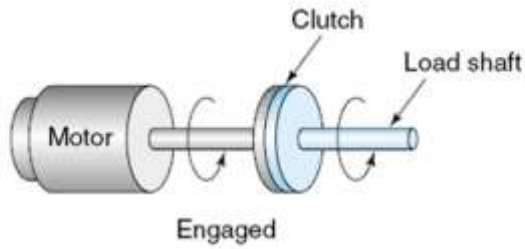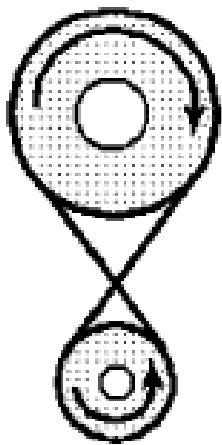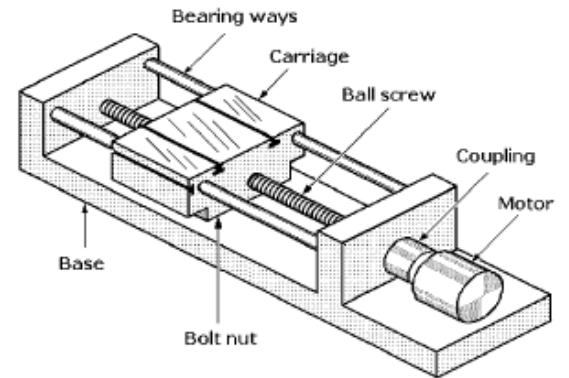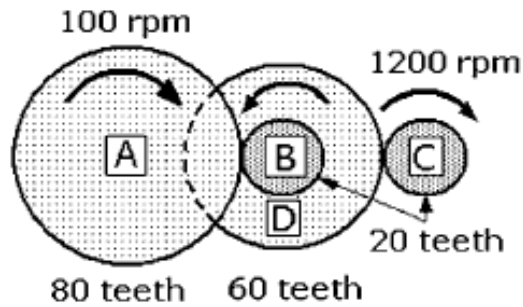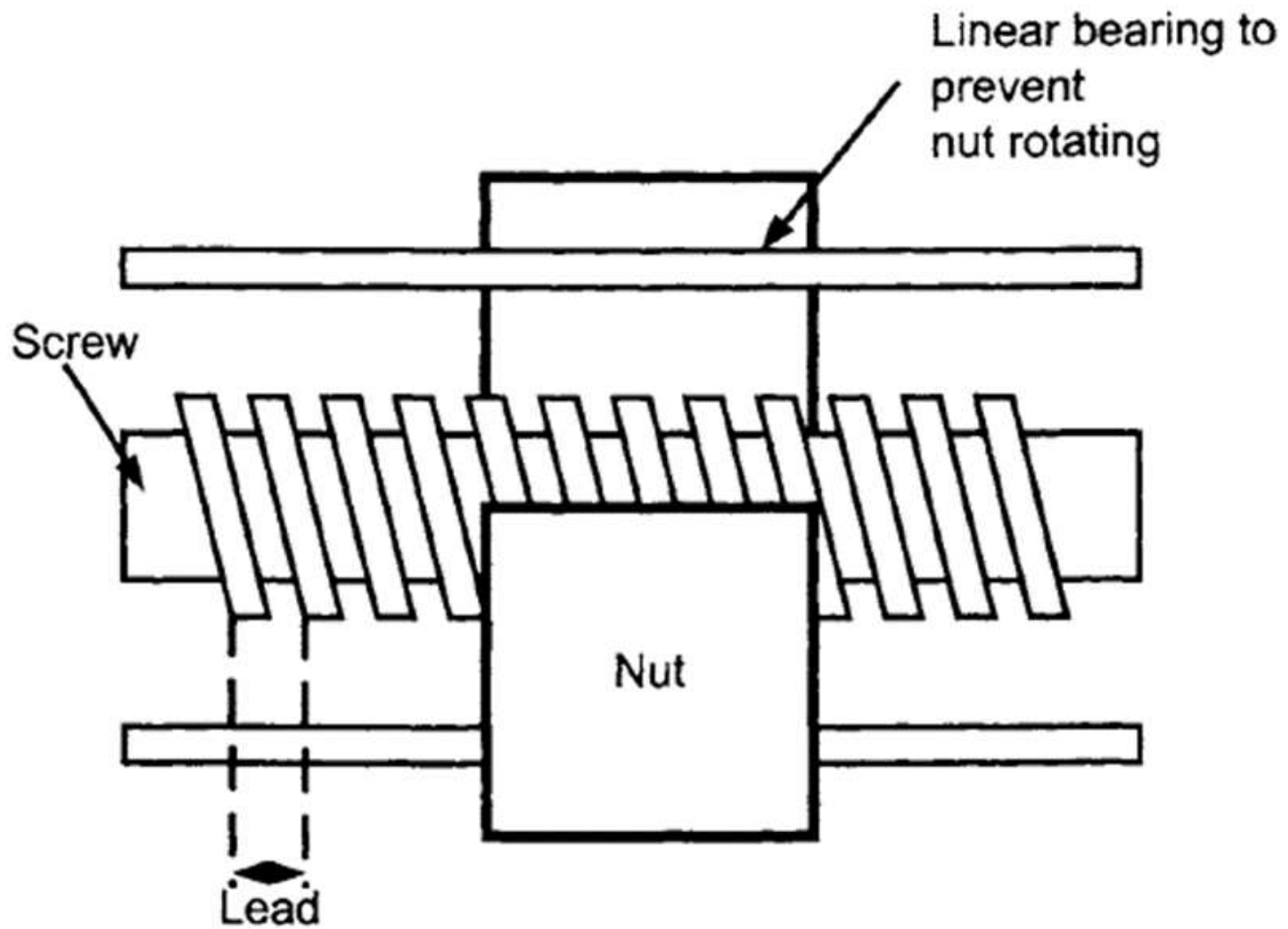
Flexible coupling
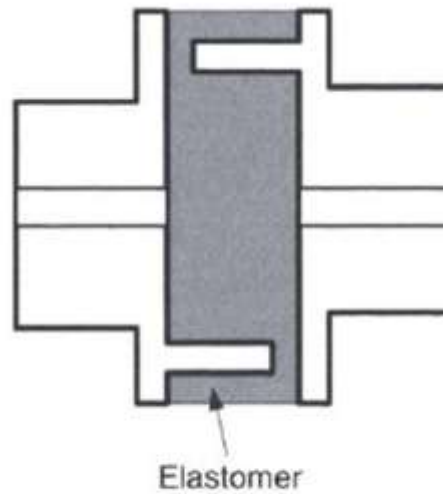
Elastomer

(a) Flexible elastomer coupling.

Spring steel bellows
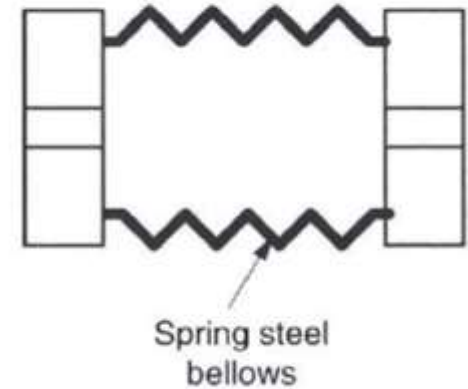
(b) Metallic bellows coupling.

Hollow shaft | axial | angled | end-to-end hollow shaft not possible

Type of gearbox	Helical gearbox (Solid shaft and shaft mounted)	Bevel gearbox and worm gearbox	Planetary gearbox
Type of shaft			
Solid shaft			
	coaxial	angled	coaxial

Gear system-motor connections

$$r_2 > r_1 \implies \omega_1 > \omega_2$$

$$N_1$$

$$\theta_1, \omega_1$$

$$T_1$$

$$r_2$$

$$T_2$$

$$\omega_2, \theta_2$$

$$N_2$$

$$\omega_1$$

$$r_1$$

$$V$$

$$\omega_2$$

$$r_2$$
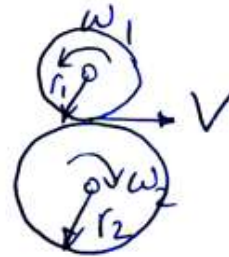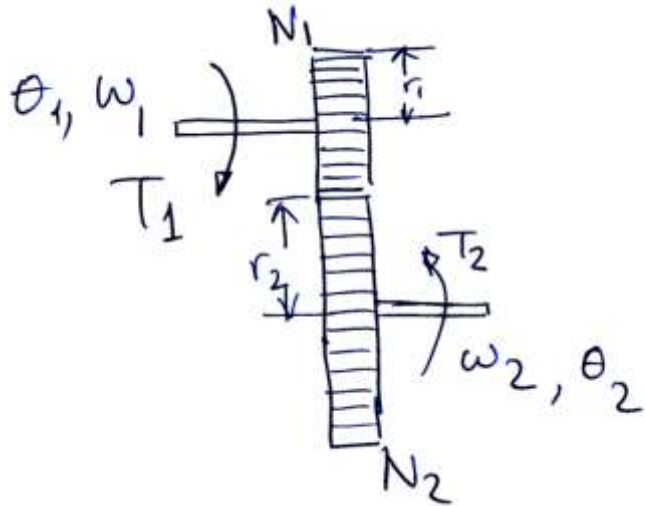
Ideal gear→No loss assumption

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}$$

$$\omega_2 = \frac{r_1}{r_2} \omega_1$$

$$r_1 \cdot \omega_1 = r_2 \cdot \omega_2 = v$$

$$T_1 \omega_1 = P_1 = T_2 \cdot \omega_2$$

$$T_1 \omega_1 = T_2 \omega_2 \longrightarrow$$

$$\frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}$$

$$\frac{r_2}{r_1} = n \succ 1 \Rightarrow T \uparrow, \omega \downarrow$$

First, let's understand the fundamental arrangement of a gear system.

In a typical setup, the first side of the gear system is driven by a servo motor.

This motor is the source of input energy and control for the system.

On the second side, we have the mechanism that needs to be moved or controlled - this is what we refer to as the 'load'.

It's crucial to recognize that from the motor's perspective, this load can be considered as the motor's 'burden' or resistance to its motion.

Now, let's dive deeper into the dynamics of this interaction. One of the key concepts in gear systems is the gear ratio, defined as the ratio of the number of teeth on the driving gear to the number of teeth on the driven gear.

This ratio is pivotal because it determines how the torque and speed from the motor are translated to the load.

A low-torque, high-speed motor can be used to drive high-torque, low-speed mechanisms effectively through the right gear ratio.

This principle is fundamental in selecting the appropriate motor and gear combination for any given application.

Consider a system where the load consists of a mass connected to a spring and experiencing viscous friction.

When a motor is connected to this load through a gear system, all the forces acting on the motor shaft are influenced by the gear ratio.

It is important to analyze these effects to understand the system's behavior fully.

For example, in our system, let's symbolize all the torque effects on the load side with the symbol T2.

By examining T2, we can gain insights into how the load's characteristics – its mass, springiness, and friction – interact with the motor through the gear system.

This analysis is crucial for predicting the system's performance and for designing a system that can handle the desired tasks efficiently and effectively.
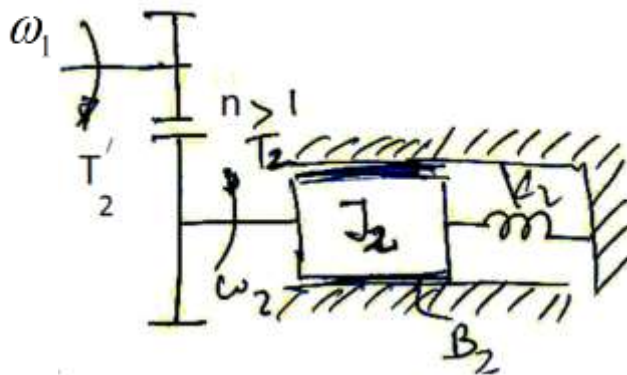
$$T_2{}' = \frac{T_2}{n}$$

$$\omega_2{}' = \omega_2 n$$

$$\frac{1}{2}J_2'\omega_1^2 = \frac{1}{2}J_2\omega_2^2 \Rightarrow$$

$$J_2' = J_2\left(\frac{\omega_2}{\omega_1}\right)^2 = \frac{J_2}{n^2}$$
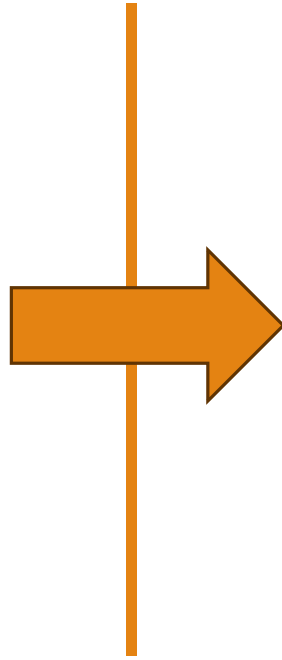
$$B_2' = \frac{B_2}{n^2}$$

$$K_2' = \frac{K_2}{n^2}$$

Energy Equivalence Principle

$$T_2 = J_2 \dot{\omega}_2 + B_2 \omega_2 + K_2 \theta_2$$

$$\theta_1 = n\theta_2 \,; \omega_1 = n\omega_2 \,; \dot{\omega}_1 = n\dot{\omega}_2$$

$$T_1 = \frac{T_2}{n}$$

$$T_2 = J_2 \dot{\omega}_2 + B_2 \omega_2 + K_2 \theta_2$$

$$T_1 = \frac{J_2}{n} \dot{\omega}_2 + \frac{B_2}{n} \omega_2 + \frac{K_2}{n} \theta_2$$

$$T_1 = \underbrace{\frac{J_2}{n^2}}_{J_2'} \dot{\omega}_1 + \underbrace{\frac{B_2}{n^2}}_{B_2'} \omega_1 + \underbrace{\frac{K_2}{n^2}}_{K_2'} \theta_1$$

$$T_L \Rightarrow on\_motor\_shaft : \frac{T_L}{n}$$

$$J_L, B_L, K_L \Rightarrow on\_motor\_shaft : \frac{J_L}{n^2}, \frac{B_L}{n^2}, \frac{K_L}{n^2}$$
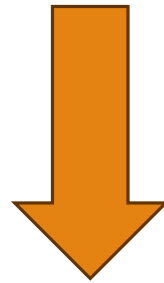
$$T_{em} = \left(J_m + \frac{J_L}{n^2} \right) \frac{d\omega_m}{dt} + \frac{T_L}{n}$$

Gear Reducer
30:1
85% Efficiency

Output
0.634 kW (0.746 kW x 0.85)
103.73 Nm (76.5 lb-ft)
58.3 RPM

Input
0.746 kW
4.068 Nm (3 lb-ft)
1750 RPM
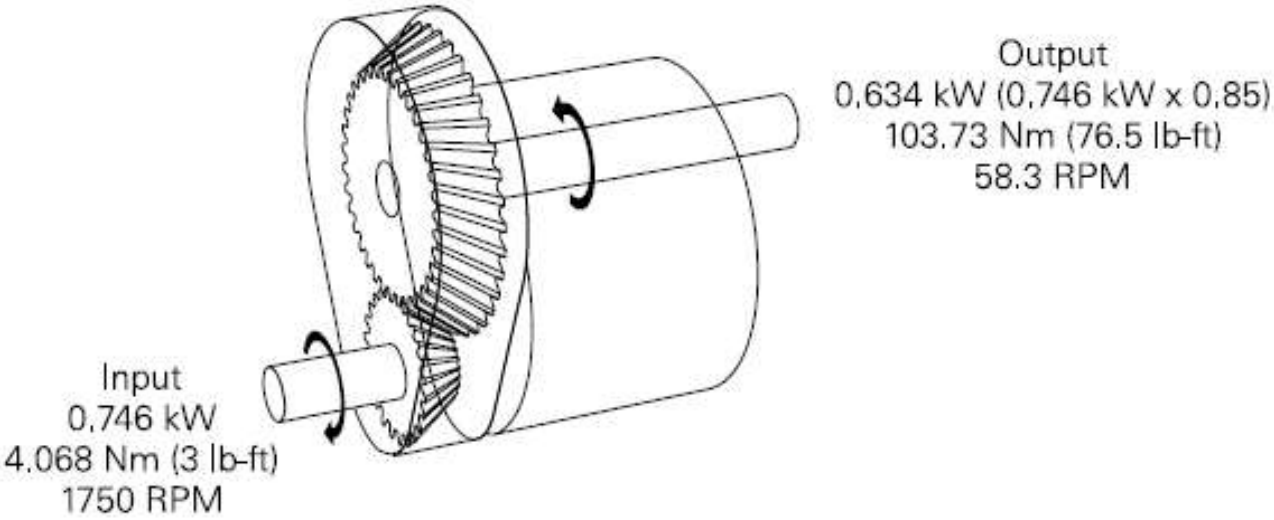
Non-ideal gear system

Belt-pulley system

Another similar system is the one in which chains are used instead of joints.
Their efficiency is approximately 80%.
To eliminate the slipping effect that may occur between the belt and the pulley, the contacting surfaces of the pulley and the belt mechanism are formed as in the gear mechanism.

Motor
θ_m , ω_m , J_m , T_m

Timing Belt

D_m

D_L

1 rev = 2π (rad)

θ = angular distance (rad)

ω = angular velocity (rad/s)

J = moment of inertia
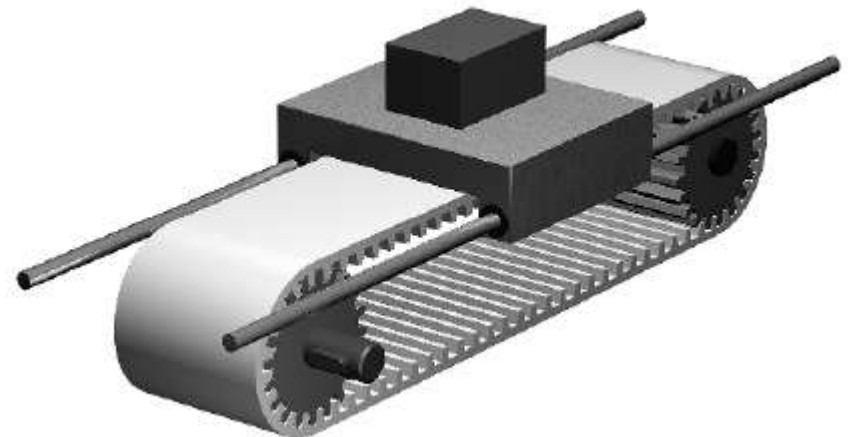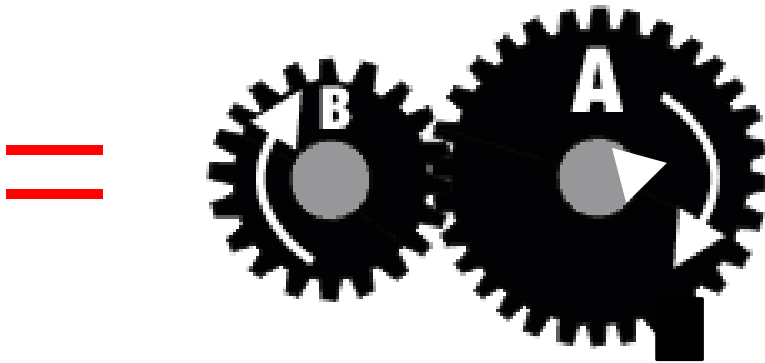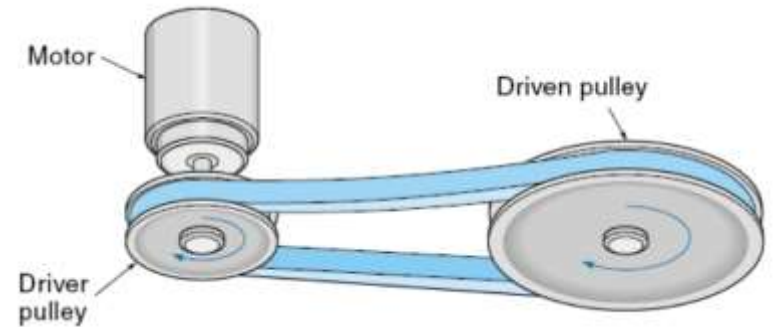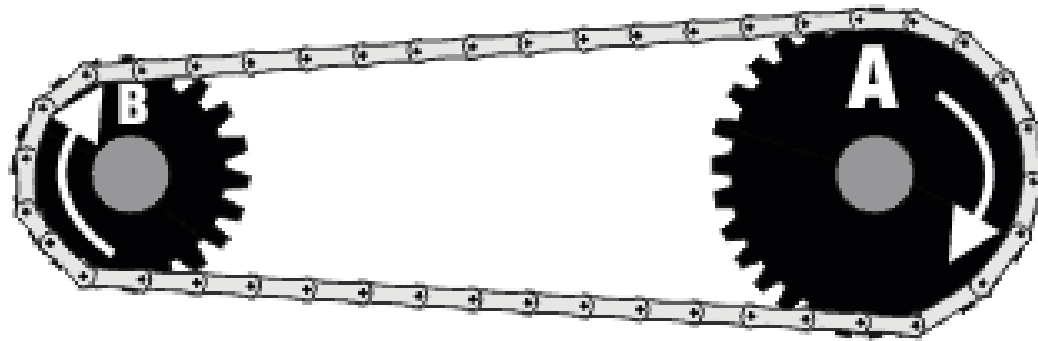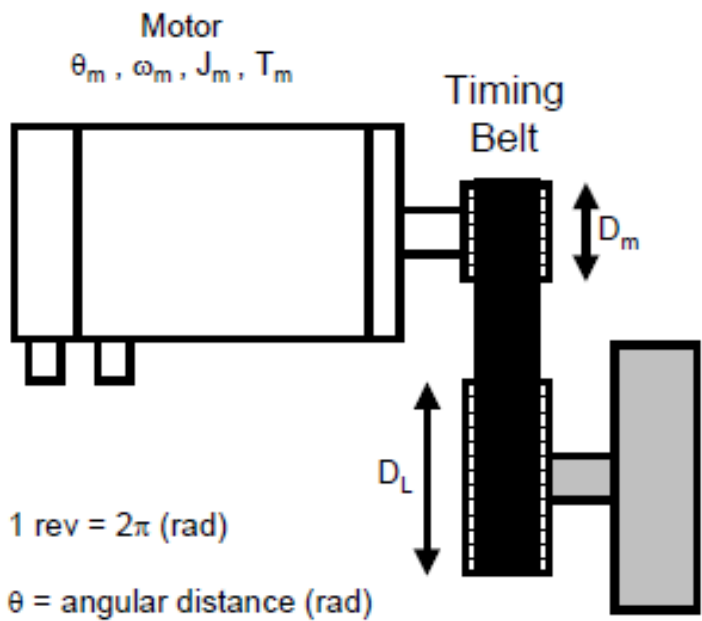
T = torque (lb-in)

Ratio = N = $\dfrac{\text{motor velocity}}{\text{load velocity}}$ = $\dfrac{D_L}{D_m}$

$\theta_m = N\,\theta_L$, $\omega_m = N\,\omega_L$

Total Inertia = $J_{total} = J_m + \dfrac{1}{N^2}\,J_L$

Load Torque Reflected to Motor = $\dfrac{1}{N}\,T_L$

Load
θ_L , ω_L , J_L , T_L

Similar expressions in gear and belt-pulley system

Motor
θ_m , ω_m , J_m , T_m

Gearing

D_m

D_L
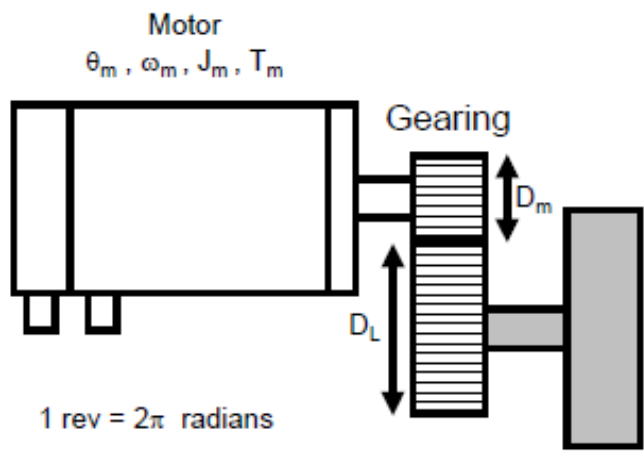
1 rev = 2π radians

Ratio = N = $\dfrac{\text{motor velocity}}{\text{load velocity}}$ = $\dfrac{N_{teeth(load)}}{N_{teeth(motor)}}$

$\theta_m = N\,\theta_L$, $\omega_m = N\,\omega_L$

Total Inertia = $J_{total} = J_m + \dfrac{1}{N^2}\,J_L$

Load Torque Reflected to Motor = $\dfrac{1}{N}\,T_L$

Load
θ_L , ω_L , J_L , T_L
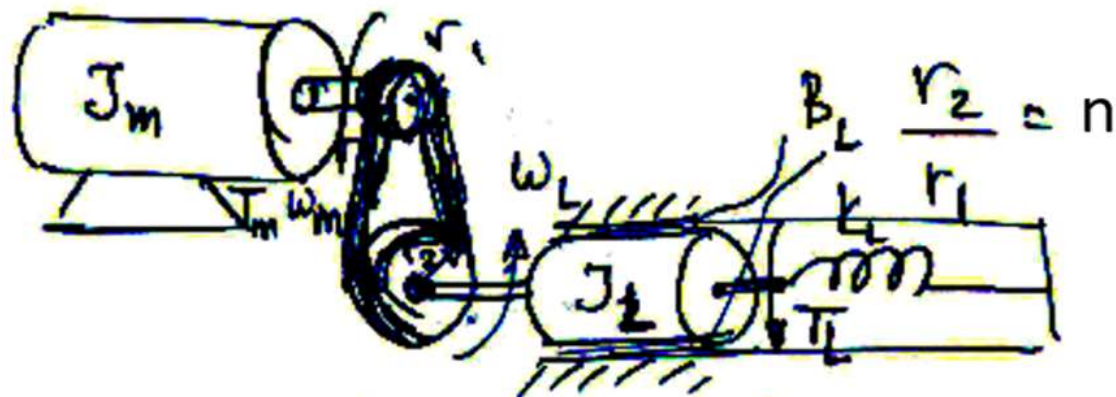
$$\left(T_m - J_m \frac{d\omega_m}{dt} \right) \cdot n = J_L \frac{d\omega_L}{dt} + T_L + B_L \omega_L + K_L \theta_L$$

$$T_m = \left(J_m + \frac{J_L}{n^2} \right) \frac{d\omega_m}{dt} + \frac{B_L}{n^2} \omega_m + \frac{K_L}{n^2} \theta_m + \frac{1}{n} T_L$$
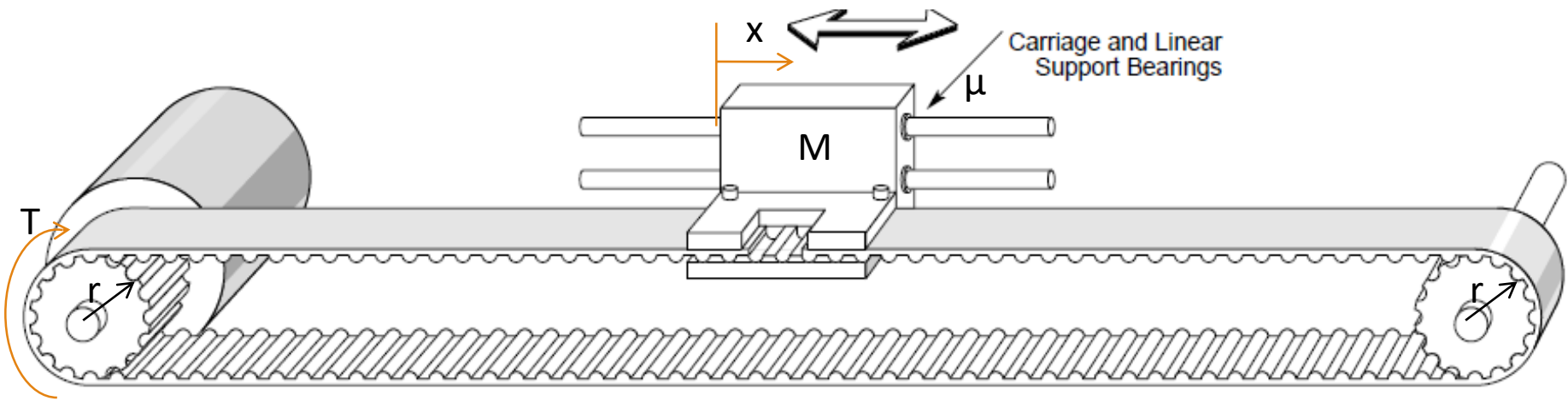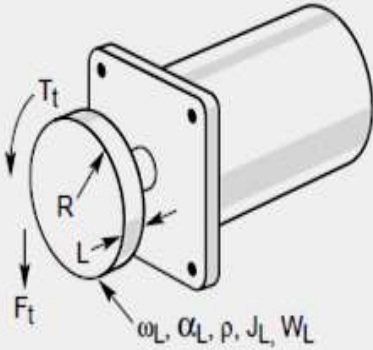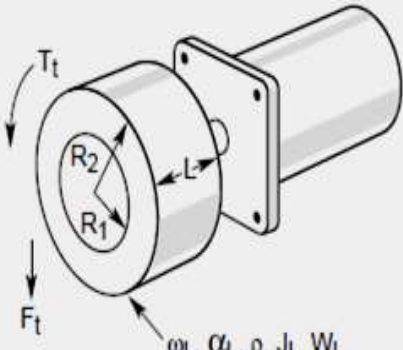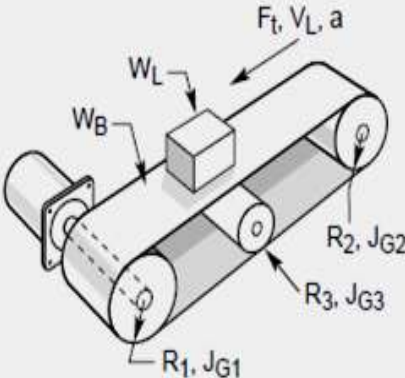
$$T - J_m \frac{d\omega}{dt} - B_m \omega = Fr$$

$$T = r\,M\,a + M\,\mu\,g\,r + J_m \frac{d\omega}{dt} + B_m \omega$$

$$= \left(J_m + M\,r^2\right)\frac{d\omega}{dt} + B_m\,\omega + M\,\mu\,g\,r$$

Sample effects created by various mechanisms connected to the motor shaft

	Inertia Solid Cylinder	Inertia Hollow Cylinder	Tangential (Conveyor, Rack and Pinion, etc.)
ω	$\omega = \omega_L$	$\omega = \omega_L$	$\omega = \dfrac{V_L}{R_1}$
α	$\alpha = \alpha_L$	$\alpha = \alpha_L$	$\alpha = \dfrac{a}{R_1}$
J_T	$W_L = \pi L \rho R^2$ $J_T = J_L + J_{Motor}$ $J_L = \frac{1}{2} \dfrac{W_L}{g} \cdot R^2$	$W_L = \pi L \rho [R_2^2 - R_1^2]$ $J_T = J_L + J_{Motor}$ $J_L = \frac{1}{2} \dfrac{W_L}{g} [R_2^2 + R_1^2]$	$J_{RL} = \dfrac{W_L + W_B}{g} \cdot R_1^2$ $J_T = J_{Motor} + J_{RL} + J_{G1} + J_{G2} \left(\dfrac{R_1}{R_2}\right)^2 + J_{G3} \left(\dfrac{R_1}{R_3}\right)^2$
T_{RL}	$F_t = F_{Friction} + F_{Applied} + F_{Gravity}$ $T_{RL} = F_t \cdot R + T_t$	$F_t = F_{Friction} + F_{Applied} + F_{Gravity}$ $T_{RL} = F_t \cdot R_2 + T_t$	$F_t = F_{Friction} + F_{Applied} + F_{Gravity}$ $T_{RL} = \dfrac{F_t \cdot R_1}{e}$

First, let's consider the transmission mechanism in electric vehicles. At its core, this mechanism is essentially a simple reduction gear set. Unlike conventional internal combustion engine vehicles, electric cars do not require components like a clutch to disengage the motor from the transmission system. This is because electric motors do not run when the vehicle is stationary and can change the direction of the vehicle's movement by simply reversing the motor's rotation.

A key feature of electric motors is their ability to generate high torque from zero RPM and maintain it across a large portion of their operating range. This characteristic plays a significant role in allowing vehicles to accelerate from zero to their designed maximum speed with a single reduction ratio. The simplicity of the reduction gear design is advantageous, as it minimizes losses and maximizes efficiency.

Maximum Motor RPM: The peak rotational speed of the motor influences the reduction ratio required.

Tire Size: The diameter of the tires used on the vehicle affects the distance covered per revolution and, consequently, the gearing required.

Optimal Motor Efficiency Range: Motors have a specific RPM range where they operate most efficiently, which must be aligned with the reduction gearing.

Target Maximum Speed: The desired top speed of the vehicle dictates the final drive ratio.

To illustrate, let's consider an example with specific parameters:

Tire size: 245/45-17, translating to a diameter of 65.2297 cm.
Distance covered per tire revolution: 204.9247 cm.
Revolutions per kilometer: 487.9842.
Target maximum speed: 160 km/h, which is 2.6667 km/min.
Target maximum motor RPM: 9000 RPM.
Revolutions per minute for the target maximum speed (160 km/h): 1301.2912 RPM.
Required reduction ratio: 6.9162.

These calculations show how the reduction ratio is determined based on the interplay of these variables. The reduction ratio of 6.9162 in this example is critical to achieving the desired performance characteristics of the electric vehicle.

In our exploration of mechanical transmissions in servo systems, it is imperative to consider the integration of mechanical transmission design with servo system requirements. This integration is critical for optimal system performance.

Let's discuss five key requirements that must be addressed in the design of mechanical transmissions for servo systems.

1. **High Stiffness**: The stiffness of the mechanical transmission is paramount. It must be high enough to place the resonant frequency outside the servo system's bandwidth. This ensures high accuracy in position control of the load. Special attention must be given to the weakest link in the transmission system, often a component like a coupling. A single element with inadequate stiffness can compromise the integrity of the entire system.

2. **High Resonant Frequency**: The mechanical system's resonant frequencies, determined by the interplay of springs and masses, are crucial. The design and selection of mechanical elements must account for inertia, mass, and stiffness. The goal is to ensure that the lowest resonant frequency is outside the servo system's required response range. Failure to do so can lead to stability issues and unwanted vibrations.

3. Sufficient Damping: Damping in a mechanical system acts similarly to a car's shock absorber. Adequate damping can significantly enhance stability and accuracy. It helps to mitigate rapid oscillations and smoothens the system's response to disturbances.

4. Linear Transfer Function Between Motor and Load: In most systems, load control is indirect, with position feedback occurring at the motor shaft. Thus, it's essential for mechanical transmission to minimize backlash, reversing errors, and lost motion between the motor and load. Nonlinearities, along with excessive speed-dependent friction, can lead to system instabilities and hunting phenomena, where the system continually oscillates around a setpoint.

5. Low Inertia of Moving Parts: AC servo systems often employ low rotor inertia motors for enhanced responsiveness. The load inertia reflected to the motor shaft should ideally be less than a 5:1 ratio, with 1:1 being ideal. The selection of transmission elements is critical in reducing reflected inertia. Given that AC servo motors can operate at high speeds (e.g., 4500 RPM), using speed reduction can capitalize on the available speed range of the servo system. Additionally, the choice of diameter and materials for components like couplings and pulleys is crucial in minimizing overall system inertia.

$$r\theta = x$$

$$\omega r = v$$
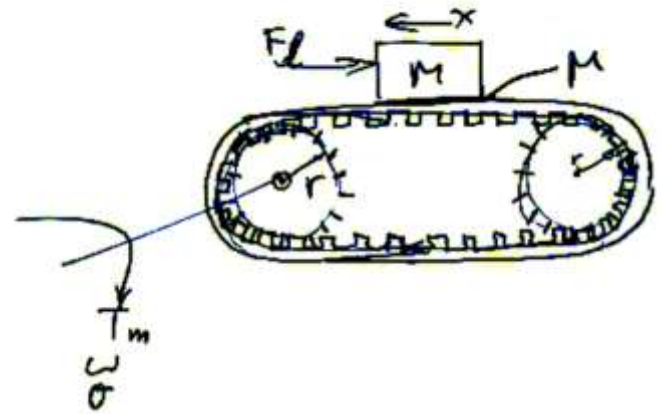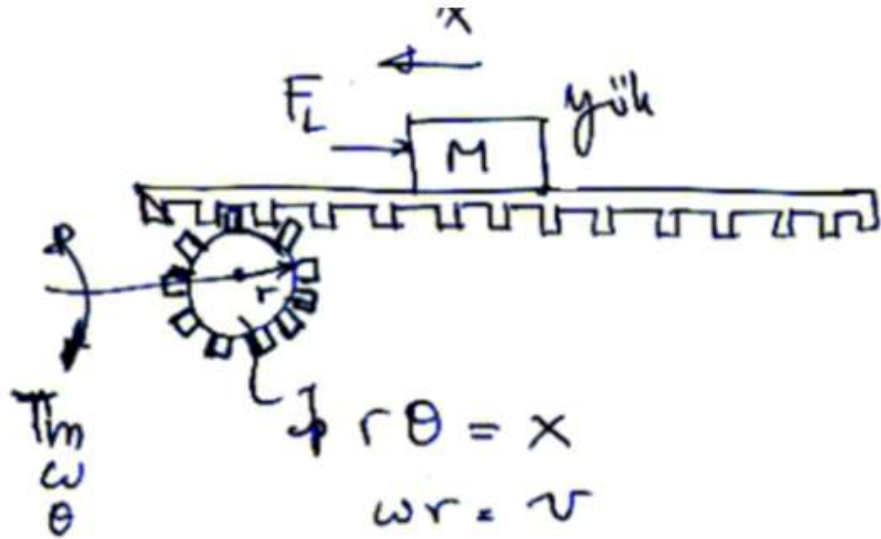
$$J = J_p + Mr^2$$

$$T_m = J_m \frac{d\omega}{dt} + J \frac{d\omega}{dt} + F_L r$$

$$T_m = \left(J_m + Mr^2\right) \frac{d\omega}{dt} + F_L r$$

Motor − Driven Double Pulleys

$$M_{p1} = M_{p2} = M_p$$

$$r_{o1} = r_{o2} = r_o$$

$$r_{i1} = r_{i2} = r_i$$

$$J_{p1} = J_{p2} = J_p = \frac{M_p \left(r_0^2 + r_i^2 \right)}{2}$$

$$T_m = J_m \frac{d\omega_m}{dt} + 2J_p \frac{d\omega_m}{dt} + F_{fr} r_o$$

g = gravity constant = 386 in/s^2

W_{total} = weight (lbs) = W_{load} + weight of moving parts

v = linear velocity (in/min)

x = linear distance (in)

$\theta_L = x/r$, $\omega_L = \dfrac{v}{60\,r}$, n (r.p.m.) $= \dfrac{v}{2\pi r}$

$F_{gravity} = W_{total}\sin\theta$, $F_{friction} = \mu\,W_{total}\cos\theta$

$T_L = F_{total}\,r + T_{acc}$, $T_{acc} = \left[J_{pulleys/pinion} + \dfrac{W_{total}}{g} r^2 \right] \dfrac{dn}{dt}$

$F_{total} = F_{external} + F_{friction} + F_{gravity}$

⟵ $F_{external}$

⟵ $F_{gravity}$

⟶ $F_{friction}$

Load (W_{load})

r = drive radius (in)

Incline angle

θ

Motor
θ_m , ω_m , J_m , T_m

N:1

Optional Timing Belt or Gear Reducer

Load Side
θ_L , ω_L , J_L , T_L
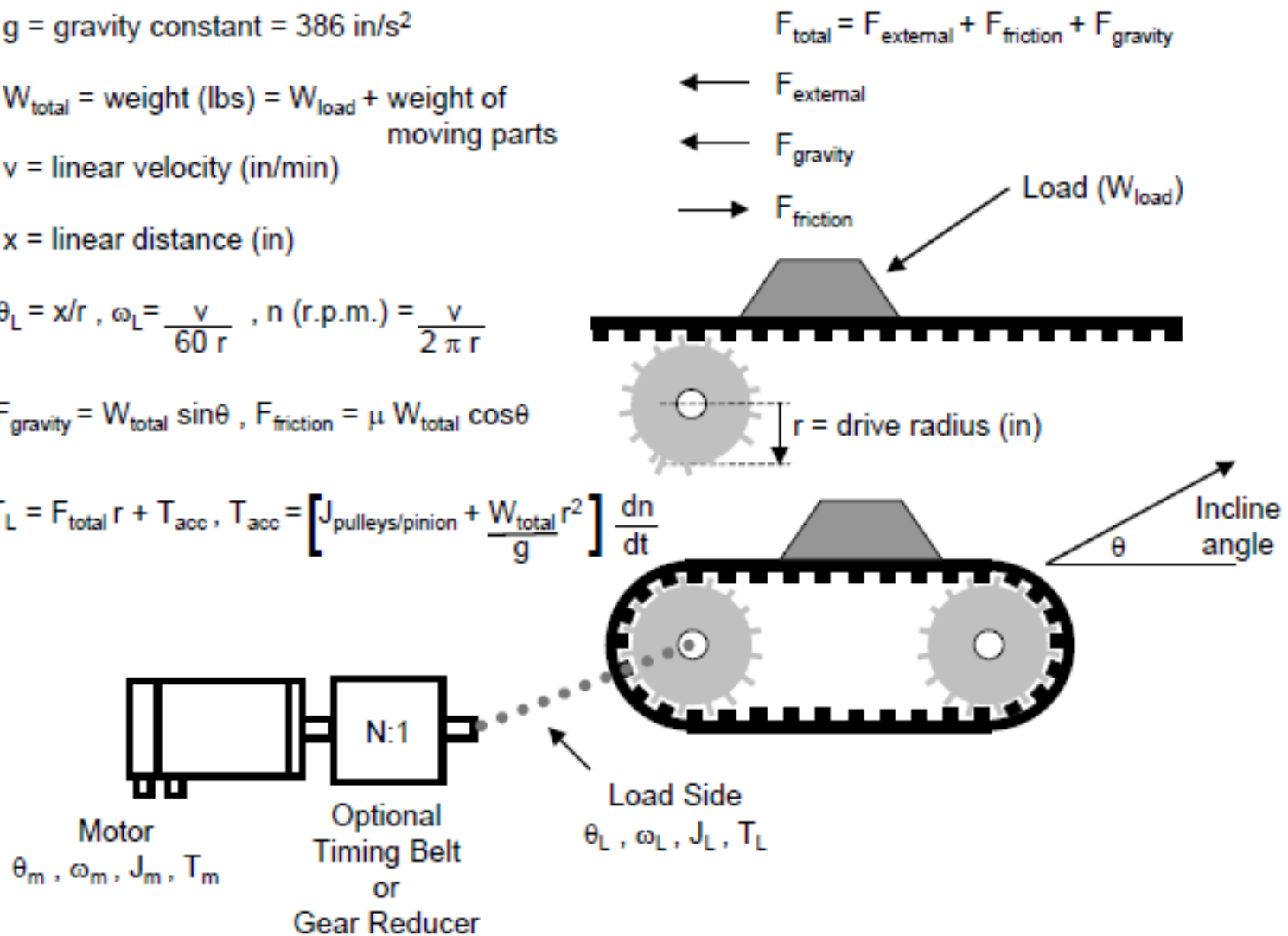
Flexible coupling elements are used to connect the shafts of servomotors to mechanical coupling elements and mechanisms.

The figure below shows the flexible coupling used to connect the shafts.
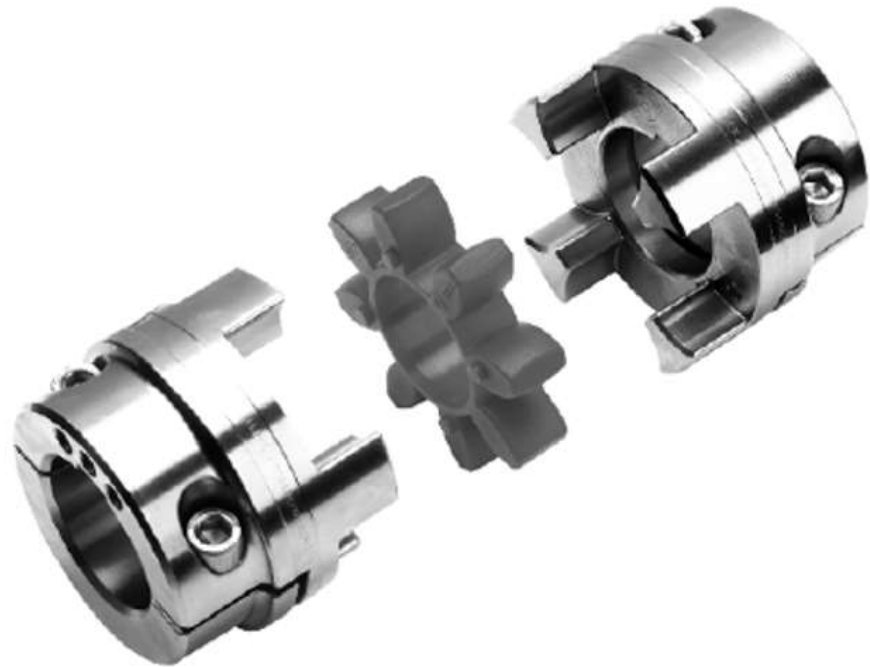
Backlash effect between teeth in gear mechanisms, flexible couplings, screw systems and time belt systems

Backlash
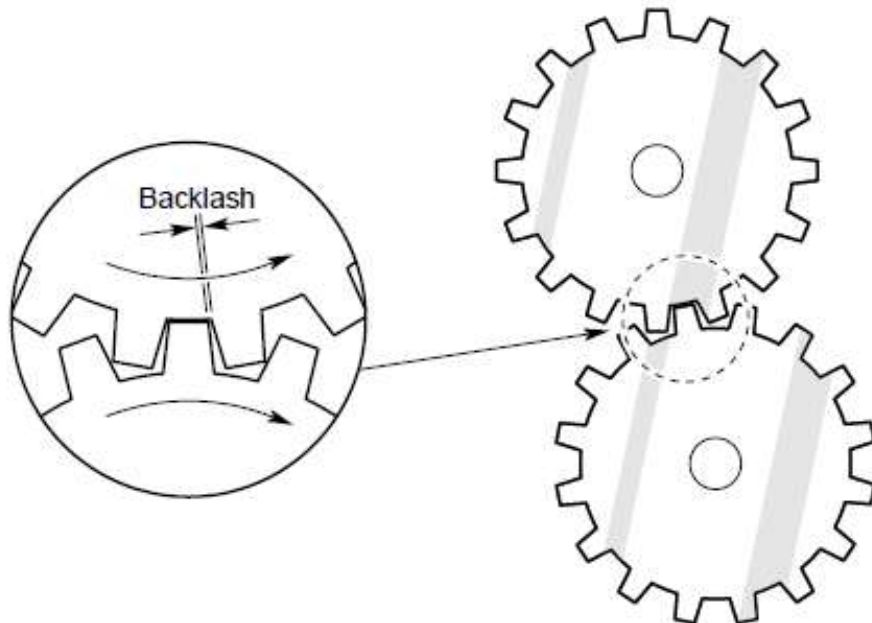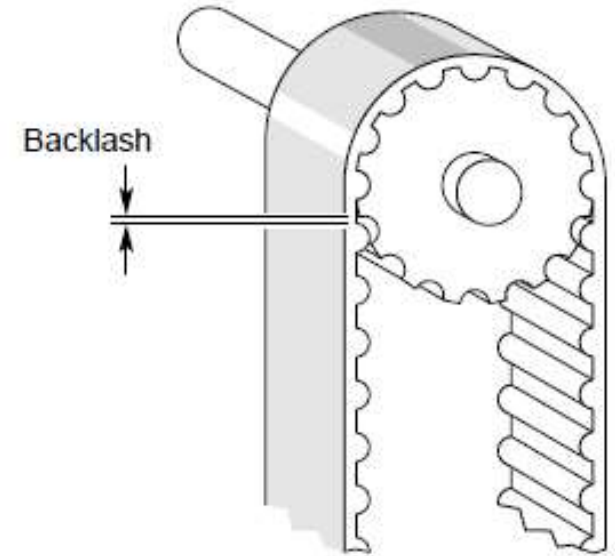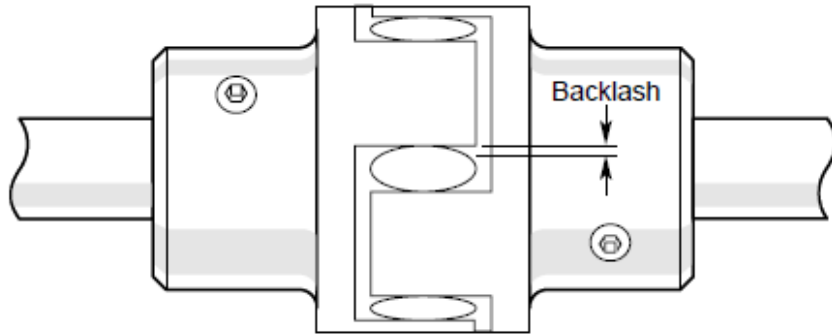
Backlash

Backlash

Primary Sources of Backlash

1. Drive Nut/Leadscrew Assembly

Backlash

Ballnut
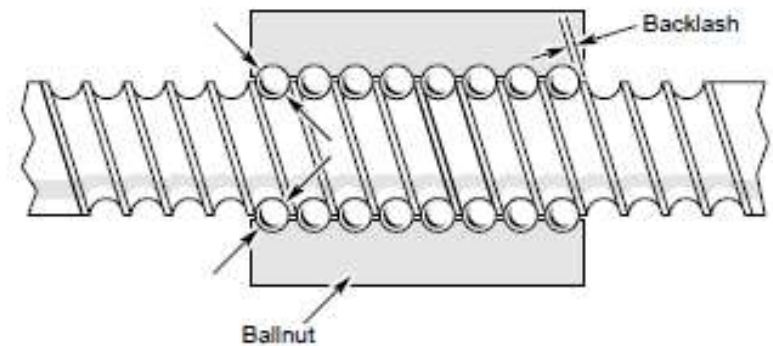
Gear Mechanisms

Definition & Significance: Backlash in gears is defined as the clearance or lost motion in a mechanism caused by gaps between the teeth. It's a critical parameter in gear design, affecting precision and noise levels.

Causes and Effects: This phenomenon occurs due to imperfections in tooth geometry, wear, or intentional design for thermal expansion allowance. Excessive backlash leads to jarring and vibration, reducing the lifespan of the gears and potentially leading to failure in precision applications.

Flexible Couplings

Overview: In flexible couplings, the backlash is less about tooth interaction and more about the elastic deformation of the coupling material.

Implications: While some backlash is necessary for tolerating misalignments and thermal expansion, excessive play can result in angular misalignment, torsional vibrations, and reduced transmission efficiency.

Screw Systems

Characteristics: In screw systems, the backlash is the axial free movement of the screw shaft relative to its nut. It's a critical factor in precision linear motion applications.

Management Strategies: Backlash in screw systems can be managed through preloading, using double nuts, or designing for tighter tolerances. However, this might increase wear and require more maintenance.

Timing Belt Systems

Basics: Backlash in timing belt systems refers to the slack or free play between the belt teeth and the pulley grooves.

Consequences and Control: It can lead to timing inaccuracies, affecting the synchronization of mechanical components. This is often controlled through tensioning mechanisms and precise pulley-to-belt matching.

General Strategies for Mitigating Backlash

Design Considerations: Optimal design for minimal clearance without causing binding or excessive wear.

Material Selection: Using materials with appropriate properties to reduce wear and thermal expansion issues.

Precision Manufacturing: Ensuring high manufacturing tolerances to reduce the need for excessive clearances.

Regular Maintenance: To identify and rectify wear-related increases in backlash.

Conclusion

Understanding and managing backlash is crucial in mechanical design for ensuring performance, durability, and precision. It requires a balanced approach considering design, material, manufacturing, and maintenance aspects.

Example: In the system below, T_m is the input and θ_y is the output.

1) Obtain transfer function model
2) Obtain the state space model
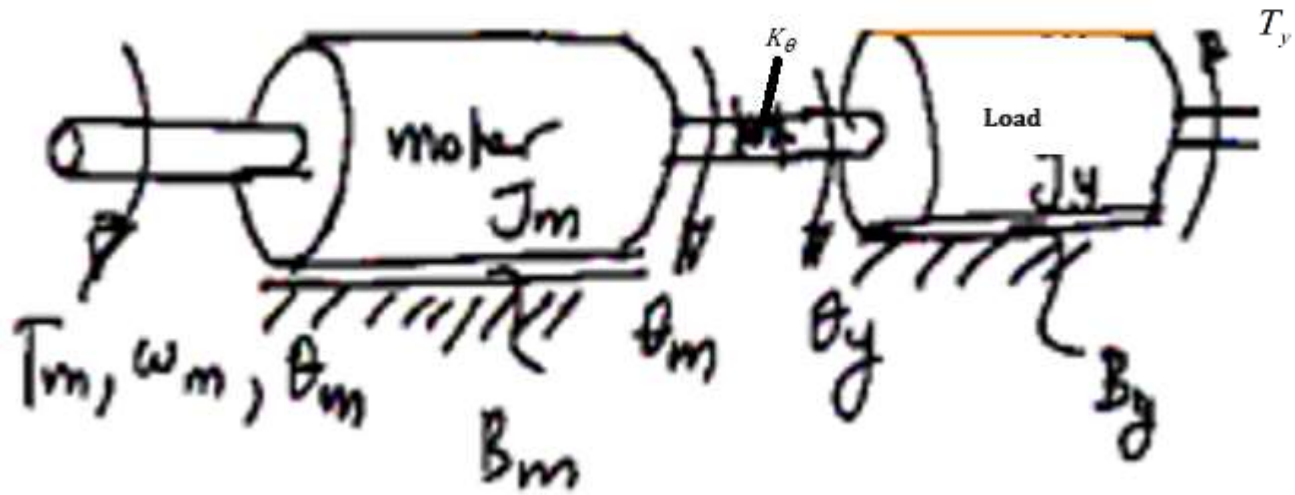
$$T_m = J_m \frac{d\omega_m}{dt} + B_m \omega_m + K\left(\theta_m - \theta_y\right)$$

$$K\left(\theta_m - \theta_y\right) = J_y \frac{d\omega_y}{dt} + B_y \omega_y + T_y$$

$$T_m(s) = \left(J_m s^2 + B_m s + k \right) \theta_m(s) - k\theta_y(s)$$

$$k\left[\theta_m(s) - \theta_y(s) \right] = \left(s^2 J_y + s B_y \right) \theta_y(s) + T_y(s)$$

$$\boxed{k\theta_m(s) = \left(s^2 J_y + s B_y + k \right) \theta_y(s) + T_y(s)}$$

$$\theta_m(s) = \frac{s^2 J_y + s B_y + k}{k} \theta_y(s) + \frac{1}{k} T_y(s)$$

$$T_m(s) = \frac{\left(J_m s^2 + B_m s + k \right)\left(s^2 J_y + s B_y + k \right)}{k} \theta_y(s) - k\theta_y(s)$$

$$+ \frac{J_m s^2 + B_m s + k}{k} T_y(s)$$

$$T_m(s) = s^4 J_y J_m + s^3 (J_m B_y + J_y B_m) + s^2 (K J_m + K J_y + B_m B_y)$$

$$+ \frac{(K B_m + k B_y) s}{k} \longrightarrow \theta_y(s)$$

$$+ \frac{J_m s^2 + B_m s + k}{k} T_y(s)$$

$$T_m(s) - \frac{(J_m s^2 + B_m s + k)}{k} T_y(s) = s^4 J_y J_m \quad \theta_y(s)$$

$$T_m(s) \xrightarrow{+} \bigcirc \xrightarrow{-} \frac{k}{s^4 J_y J_m + s^3 (J_m B_y + J_y B_m) + s^2 (K J_m + k J_y + B_m B_y) + (K B_m + k B_y) s} \longrightarrow \theta_y$$

with $T_y(s)$ feeding back.
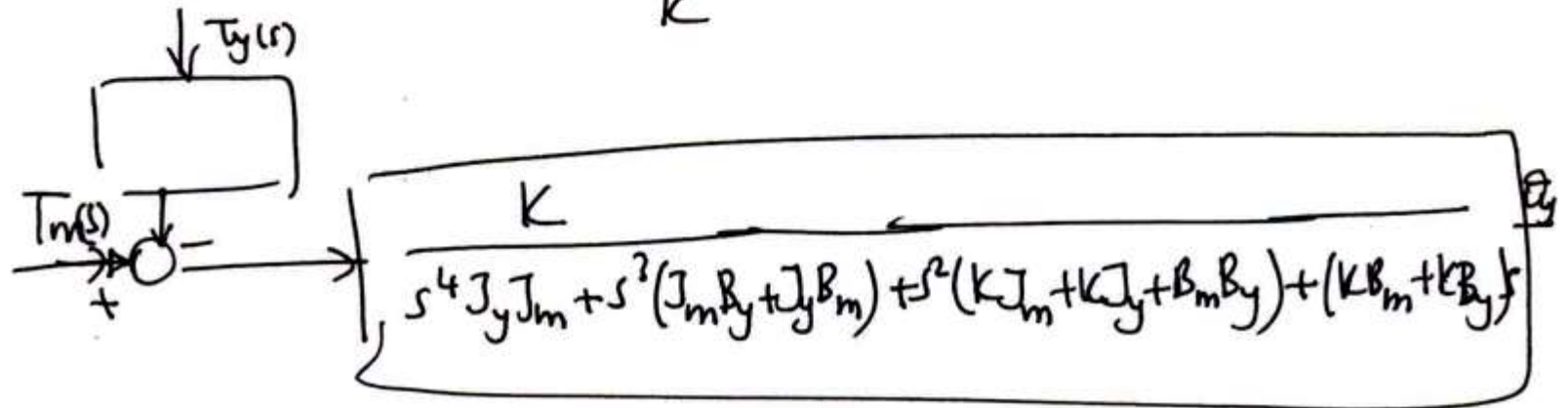
$$\frac{d}{dt}\begin{pmatrix}\theta_m\\\theta_y\\\omega_m\\\omega_y\end{pmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\-\frac{k}{J_m} & +\frac{k}{J_m} & -\frac{B_m}{J_m} & 0\\\frac{k}{J_y} & -\frac{k}{J_y} & 0 & -\frac{B_y}{J_y}\end{bmatrix}\begin{pmatrix}\theta_m\\\theta_y\\\omega_m\\\omega_y\end{pmatrix}+\begin{pmatrix}0 & 0\\0 & 0\\\frac{1}{J_m} & 0\\0 & \frac{1}{J_y}\end{pmatrix}\begin{pmatrix}T_m\\T_y\end{pmatrix}$$

$\underbrace{}_{A}$ $\underbrace{}_{B}$ similar

T_y because clock!

$$y=\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}\begin{pmatrix}\theta_m\\\theta_y\\\omega_m\\\omega_y\end{pmatrix}$$

$$\frac{d\theta_m}{dt}=\omega_m$$

$$\frac{d\omega_m}{dt}=-\frac{B_m}{J_m}\omega_m-\frac{k}{J_m}\theta_m+\frac{k}{J_m}\theta_y+\frac{1}{J_m}T_m$$

$$\frac{d\theta_y}{dt}=\omega_y$$

$$\frac{d\omega_y}{dt}=-\frac{B_y}{J_y}\omega_y+\frac{k}{J_y}\theta_m-\frac{k}{J_y}\theta_y+\frac{1}{J_y}T_y$$

$F_{external}$ $F_{total} = F_{external} + F_{friction} + F_{gravity}$

$F_{gravity}$

Load Side $F_{friction}$ Load (W_{load})

θ_L , ω_L , J_L , T_L

Table (W_{table})

Motor
θ_m , ω_m , J_m , T_m

μ = coefficient of friction)

N:1

Incline
angle

θ

Optional
Timing Belt
or
Gear Reducer

Ball Nut   Ball Screw (J_{screw})
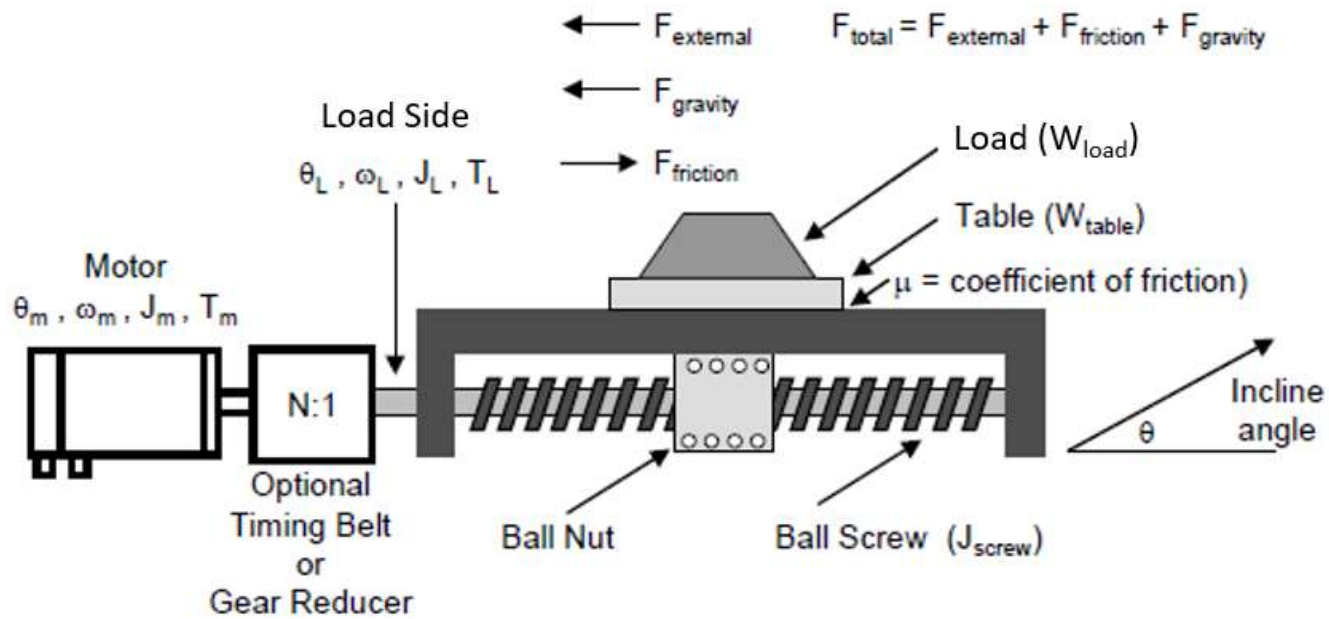
P = pitch = revs/ m of leadscrew

$\theta_L = 2 \pi P x$, $\omega_L = \dfrac{2 \pi P v}{60}$

h = lead =   m /rev of leadscrew

g = gravity constant = 9.81 m/s^2

$F_{gravity} = W_{total} \sin\theta$, $F_{friction} = \mu\, W_{total} \cos\theta$

W_{total} = weight (lbs) = $W_{load} + W_{table}$

$T_L = \dfrac{F_{total}}{2\pi P} + T_{acc}$, $T_{acc} = \left[J_{screw} + \dfrac{W_{total}}{g}(1/2\pi P)^2 \right] \dfrac{dn}{dt}$

v = linear velocity   (m/s)

x = linear distance   (m)

$$F = F_L + Mg\mu$$

$$P = T\omega = Fv$$

$$T = \frac{Fv}{\omega} = \frac{Fv}{2\pi p}$$

$$T_m = T + J_{eqv}\frac{d\omega}{dt} = \frac{F_L + Mg\mu}{2\pi p} + \frac{M}{(2\pi p)^2}\frac{d\omega}{dt} + J_m\frac{d\omega}{dt}$$

$$\dot{\theta} = \omega = 2\pi p\dot{x} = 2\pi pv$$

$$Mv^2 = J_{eqv}(2\pi pv)^2$$

$$J_{eqv} = \frac{M}{(2\pi p)^2}$$

$$T = \frac{F}{2\pi p} \qquad T_{fr} = \frac{F_{fr}}{2\pi p} = \frac{M\mu g}{2\pi p}$$

$$F_{fr} = M\mu g$$

No loss assumption

$$T\ \omega\ = F\ v$$

$$\omega = 2\pi p v \qquad T\ 2\pi p v = F\ v$$

Leadscrew

ω	$\omega = 2\pi p V_L$
α	$\alpha = 2\pi p a$
J	$J_S = 0.0012\, LD^4$ (for steel) $J_{RL} = \dfrac{W_L}{g}\left[\dfrac{1}{2\pi p}\right]^2$ $J_T = J_{RL} + J_S + J_{Motor}$
T	$F_t = F_{Friction} + F_{Applied} + F_{Gravity}$ $T_{RL} = \dfrac{F_t}{2\pi \cdot p \cdot e} + T_{PL}$

Gear Box

ω	$\omega = \dfrac{V_L}{N}$
α	$\alpha = \dfrac{\alpha_L}{N}$
J	$J_{RL} = \dfrac{J_L}{N^2}$ $J_T = J_{RL} + J_{Reducer} + J_{Motor}$
T	$F_t = F_{Friction} + F_{Applied} + F_{Gravity}$ $T_{RL} = \dfrac{(F_t \cdot R) + T_t}{N \cdot e}$

Load Calculation

Roll Load

Mechanical Configuration

Pressure

Roll

F: Tension

F : Tension (N)

V_l : Load Speed (m/min)

$1/R$: Speed Reduction Ratio

η : Mechanical Efficiency

P : Pressure

D : Roll Diameter (m)

μ : Friction Coefficient
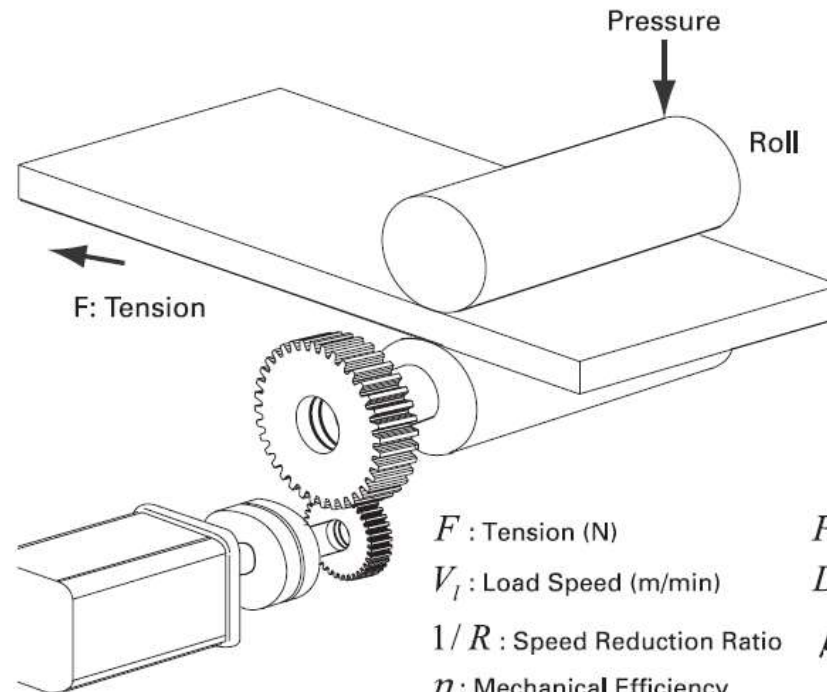
Load Torque (N· m)

$$T_L = \frac{(\mu P + F)D}{2R\eta}$$

Load Inertia Moment (Kg· m^2)

$$J_L = J_G + \frac{J_R}{R^2}$$

J_R : Roll Inertia J_G : Gear, Coupling Inertia

J$_R$

< Solid Cylinder >
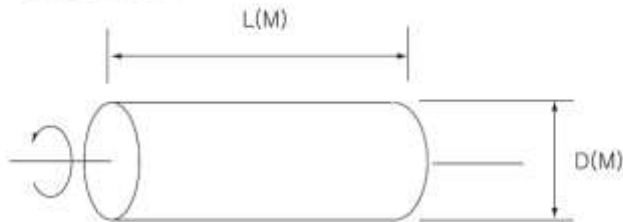
Movement Amount (M)

$$L_s = \frac{V_l}{60} \times \frac{2t_s - t_a - t_d}{2}$$

$$if \quad t_a = t_d, \quad L_s = \frac{V_l}{60} \times (t_s - t_a)$$

Motor Shaft Revolving Speed (r/min)

$$N_M = \frac{RV_l}{\pi D}$$

$$J_R = \frac{MD^2}{8} = \frac{\pi \rho L D^4}{32}$$

M : Mass[kg], ρ : Density[kg/m³]

$\rho = 7.87 \times 10^3$ [kg/ m³] : Iron

$\rho = 2.70 \times 10^3$ [kg/ m³] : Aluminum

< Hollow Cylinder >
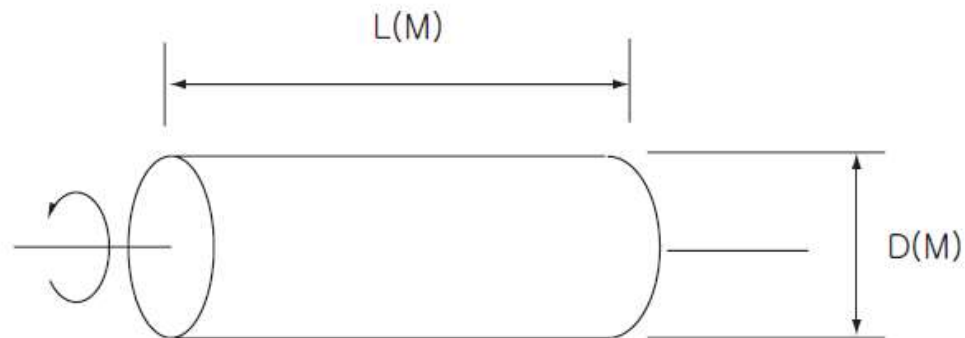
$$J_R = \frac{M(D_o^2 - D_i^2)}{8} = \frac{\pi \rho L (D_o^4 - D_i^4)}{32}$$

Minimum Acceleration Time (s)

$$t_{am} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} - T_L)}$$

J_M: Motor Inertia, T_{PM}: Motor Maximum Torque

Minimum Deceleration Time (s)

$$t_{dm} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} + T_L)}$$

Load Operation Power (W)

$$P_o = \frac{2\pi N_M T_L}{60}$$

Load Acceleration Power (W)

$$P_a = (\frac{2\pi N_M}{60})^2 \times \frac{J_L}{t_a}, \quad (t_a \leq t_{am})$$

Acceleration Torque Required (N·m)

$$T_P = \frac{2\pi N_M (J_M + J_L)}{60 t_a} + T_L, \quad (t_a \leq t_{am})$$

Torque Effective Value (N·m)

$$T_{rms} = \sqrt{\frac{T_p^2 t_a + T_L^2 (t_s - t_a - t_d) + T_s^2 t_d}{t_c}}$$

Deceleration Torque Required (N·m)

$$T_S = \frac{2\pi N_M (J_M + J_L)}{60 t_d} - T_L, \quad (t_a \leq t_{dm})$$

Timing Belt Load

Mechanical Configuration
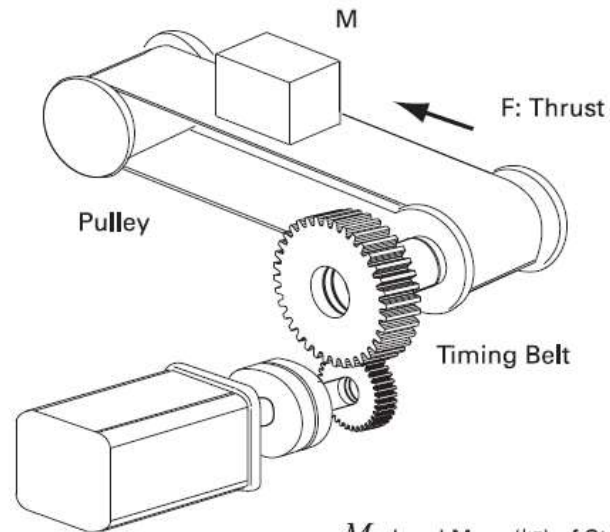
M : Load Mass (kg) of Straight Movement Part F : Thrust (N)

V_l : Load Speed (m/min) D : Pulley (m)

$1/R$: Speed Reduction Ratio μ : Friction Coefficient

η : Mechanical Efficiency

Movement Amount (m)

$$L_s = \frac{V_l}{60} \times \frac{2t_s - t_a - t_d}{2}$$

$$if \quad t_a = t_d, \quad L_s = \frac{V_l}{60} \times (t_s - t_a)$$

Motor Shaft Revolving Speed (r/min)

$$N_M = \frac{RV_l}{\pi D}$$

Load Torque (N· m)

$$T_L = \frac{(9.8\mu M + F)D}{2R\eta}$$

## Load Inertia Moment (kg·m²)

$$J_L = J_W + J_G + \frac{J_P}{R^2}$$

J_W : Load Inertia of Straight Movement Part J_P : Inertia of Pulley Part

J_G : Gear, Coupling Inertia

$$J_W = M(\frac{D}{2R})^2$$

Minimum Acceleration Time (s)

$$t_{am} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} - T_L)}$$

J_M : Motor Inertia T_{PM} : Motor Maximum Torque

Minimum Deceleration Time(s)

$$t_{dm} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} + T_L)}$$

Load Operation Power (W)

$$P_o = \frac{2\pi N_M T_L}{60}$$

Load Acceleration Power (W)

$$P_a = (\frac{2\pi N_M}{60})^2 \times \frac{J_L}{t_a}, \ \ (t_a \leq t_{am})$$

Acceleration Torque Required (N·m)

$$T_P = \frac{2\pi N_M (J_M + J_L)}{60 t_a} + T_L, \ \ (t_a \leq t_{am})$$

Deceleration Torque Required (N·m)

$$T_S = \frac{2\pi N_M (J_M + J_L)}{60 t_d} - T_L, \ \ (t_a \leq t_{dm})$$

Torque Effective Value (N·m)

$$T_{rms} = \sqrt{\frac{T_p^2 t_a + T_L^2(t_s - t_a - t_d) + T_s^2 t_d}{t_c}}$$

Horizontal Ball Screw Load   Machine Configuration

M
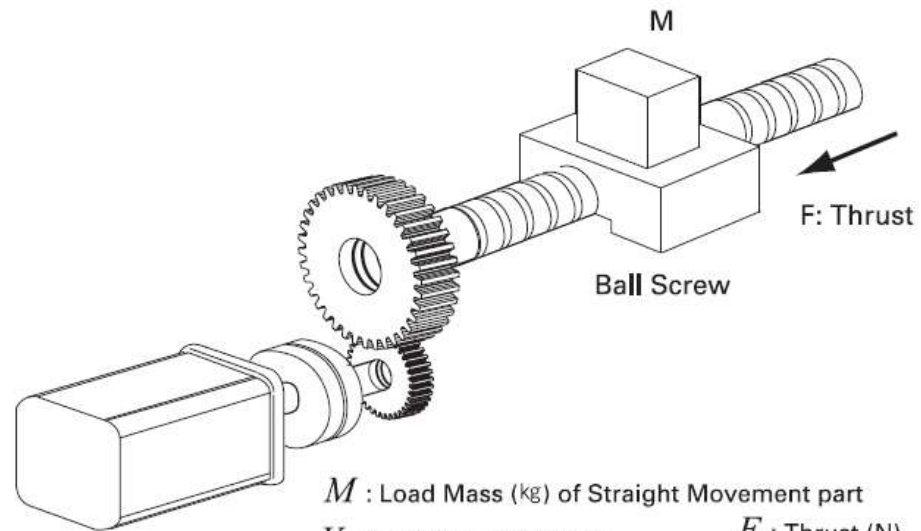
F: Thrust

Ball Screw

M : Load Mass (kg) of Straight Movement part

V_l : Load Speed (m/min)

P_B : Ball Screw Lea

L_B : Ball Screw Length (m)

η : Mechanical Efficiency

F : Thrust (N)

D_B : Ball Screw

$1/R$: Speed Reduction Ratio

μ : Friction Coefficient

Movement Amount (m)

$$L_s = \frac{V_l}{60} \times \frac{2t_s - t_a - t_d}{2}$$

$$if \quad t_a = t_d, \quad L_s = \frac{V_l}{60} \times (t_s - t_a)$$

Motor Shaft Revolving Speed (r/min)

$$N_M = \frac{RV_l}{P_B}$$

Load Torque (N· m)

$$T_L = \frac{(9.8\mu M + F)P_B}{2\pi R\eta}$$

Load Inertia Moment (kg· m²)

$$J_L = J_W + J_G + \frac{J_B}{R^2}$$

J_W : Load Inertia of Straight Movement Part J_B : Ball Screw Inertia

J_G : Gear, Coupling Inertia

$$J_W = M(\frac{P_B}{2\pi R})^2, \quad J_B = \frac{M_B D_B^2}{8} = \frac{\pi \rho L_B D_B^4}{32}$$

M_B : Ball Screw Mass[kg]

$\rho = 7.87 \times 10^3$ [kg/ m³] ; Iron

$\rho = 2.70 \times 10^3$ [kg/ m³] ; Aluminum

Minimum Acceleration Time(s)

$$t_{am} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} - T_L)}$$

J_M : Motor Inertia, T_{PM} : Motor Maximum Torque

Minimum Deceleration Time (s)

$$t_{dm} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} + T_L)}$$

Load Operation Power (W)

$$P_o = \frac{2\pi N_M T_L}{60}$$

Load Acceleration Power (W)

$$P_a = (\frac{2\pi N_M}{60})^2 \times \frac{J_L}{t_a}, \ \ (t_a \leq t_{am})$$

Acceleration Torque Required (N· m)

$$T_P = \frac{2\pi N_M (J_M + J_L)}{60 t_a} + T_L, \ \ (t_a \leq t_{am})$$

Deceleration Torque Required (N· m)

$$T_S = \frac{2\pi N_M (J_M + J_L)}{60 t_d} - T_L, \ \ (t_a \leq t_{dm})$$

Torque Effective Value (N· m)

$$T_{rms} = \sqrt{\frac{T_p^2 t_a + T_L^2 (t_s - t_a - t_d) + T_s^2 t_d}{t_c}}$$

Vertical Ball Screw Load

Mechanical Configuration

M

F: Thrust

Ball Screw

Mc
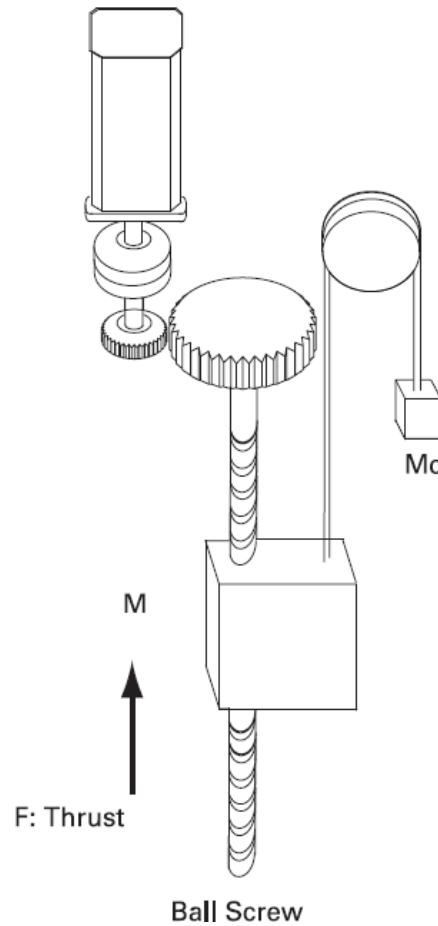
M : Load Mass (kg) of Straight Movement Pa

V_l : Load Speed (m/min)

P_B : Ball Screw Lead (m)

L_B : Ball Screw Length (m)

η : Mechanical Efficiency

F : Thrust (N)

D_B : Ball Screw Diameter (m) Ball Screw

$1/R$: Speed Reduction Ratio

μ : Friction Coefficient

M_C : Mass of Counter (kg)

Movement Amount (m)

$$L_s = \frac{V_l}{60} \times \frac{2t_s - t_a - t_d}{2}$$

$$\text{if } t_a = t_d, \quad L_s = \frac{V_l}{60} \times (t_s - t_a)$$

Motor Shaft Revolving Speed (r/min)

$$N_M = \frac{R V_l}{P_B}$$

Load Torque (N· m)

$$T_L = \frac{\{9.8\mu(M - M_C) + F\}P_B}{2\pi R \eta}$$

Load Inertia Moment (kg· m²)

$$J_L = J_W + J_G + \frac{J_B}{R^2}$$

J_W : Load Inertia of Straight Movement Part, J_B : Ball Screw Inertia

J_G : Gear, Coupling Inertia

$$J_W = (M + M_C)\left(\frac{P_B}{2\pi R}\right)^2$$

$$J_B = \frac{M_B D_B^2}{8} = \frac{\pi \rho L_B D_B^4}{32}$$

M_B :Ball Screw Mass[kg]

$\rho = 7.87 \times 10^3$ [kg/m³] : Iron

$\rho = 2.70 \times 10^3$ [kg/m³] : Aluminum

Load Acceleration Power (W)

$$P_a = (\frac{2\pi N_M}{60})^2 \times \frac{J_L}{t_a}, \ (t_a \le t_{am})$$

Acceleration Torque Required (N· m)

$$T_P = \frac{2\pi N_M (J_M + J_L)}{60 t_a} + T_L, \ (t_a \le t_{am})$$

Deceleration Torque Required (N· m)

$$T_S = \frac{2\pi N_M (J_M + J_L)}{60 t_d} - T_L, \ (t_a \le t_{dm})$$

Torque Effective Value (N· m)

$$T_{rms} = \sqrt{\frac{T_p^2 t_a + T_L^2 (t_s - t_a - t_d) + T_s^2 t_d}{t_c}}$$

Minimum Acceleration Time (s)

$$t_{am} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} - T_L)}$$

J_M: Motor Inertia, T_{PM}: Motor Maximum Torque

Minimum Deceleration Time(s)

$$t_{dm} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} + T_L)}$$

Load Operation Power (W)

$$P_o = \frac{2\pi N_M T_L}{60}$$

Rack & Pinion Load

Mechanical Configuration
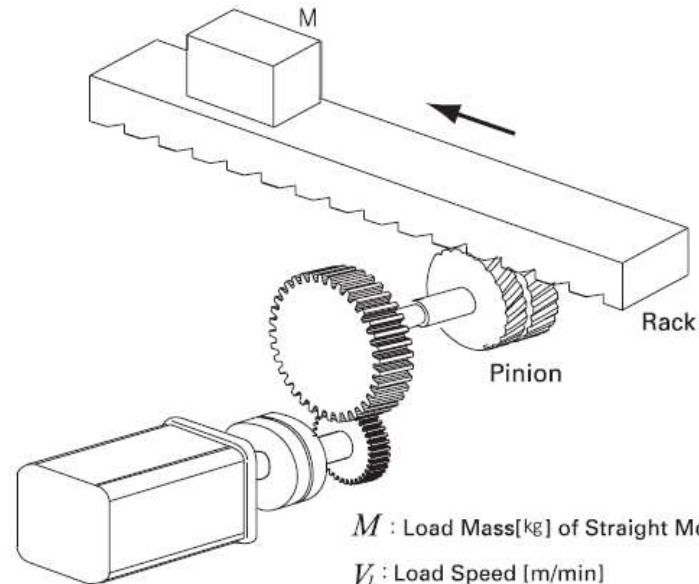
M : Load Mass[kg] of Straight Movement Part $\quad$ F : Thrust [N]

V_l : Load Speed [m/min] $\qquad\qquad\qquad\qquad$ D : Pinion Diameter [m]

l/R : Speed Reduction Ratio $\qquad\qquad\qquad$ t : Pinion Thickness [m]

η : Mechanical Efficiency $\qquad\qquad\qquad$ μ : Friction Coefficient

Movement Amount (m)

$$L_s = \frac{V_l}{60} \times \frac{2t_s - t_a - t_d}{2}$$

$$if \quad t_a = t_d, \quad L_s = \frac{V_l}{60} \times (t_s - t_a)$$

Motor Shaft Revolving Speed (r/min)

$$N_M = \frac{RV_l}{P_B}$$

Load Torque (N· m)

$$T_L = \frac{(9.8\mu M + F)D}{2R\eta}$$

Load Inertia Moment (kg· m²)

$$J_L = J_W + J_G + \frac{J_P}{R^2}$$

J_W : Load Inertia of Straight Movement Part J_P : Pinion Inertia

J_G : Gear, Coupling Inertia

$$J_W = M(\frac{D}{2R})^2, \quad J_P = \frac{M_p D^2}{8} = \frac{\pi \rho t D^4}{32}$$

M_P : Pinion Mass[kg]

$\rho = 7.87 \times 10^3$ [kg/ ㎥] : Iron

$\rho = 2.70 \times 10^3$ [kg/ ㎥] : Aluminum

Minimum Acceleration Time (s)

$$t_{am} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} - T_L)}$$

J_M : Motor Inertia T_{PM} : Motor Maximum Torque

Minimum Deceleration Time (s)

$$t_{dm} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} + T_L)}$$

Load Operation Power (W)

$$P_o = \frac{2\pi N_M T_L}{60}$$

Load Acceleration Power (W)

$$P_a = (\frac{2\pi N_M}{60})^2 \times \frac{J_L}{t_a}, \ \ (t_a \leq t_{am})$$

Acceleration Torque Required (N·m)

$$T_P = \frac{2\pi N_M (J_M + J_L)}{60 t_a} + T_L, \ \ (t_a \leq t_{am})$$

Deceleration Torque Required (N·m)

$$T_S = \frac{2\pi N_M (J_M + J_L)}{60 t_d} - T_L, \ \ (t_a \leq t_{dm})$$

Torque Effective Value (N·m)

$$T_{rms} = \sqrt{\frac{T_p^2 t_a + T_L^2 (t_s - t_a - t_d) + T_s^2 t_d}{t_c}}$$

Disk Load

Mechanical Configuration
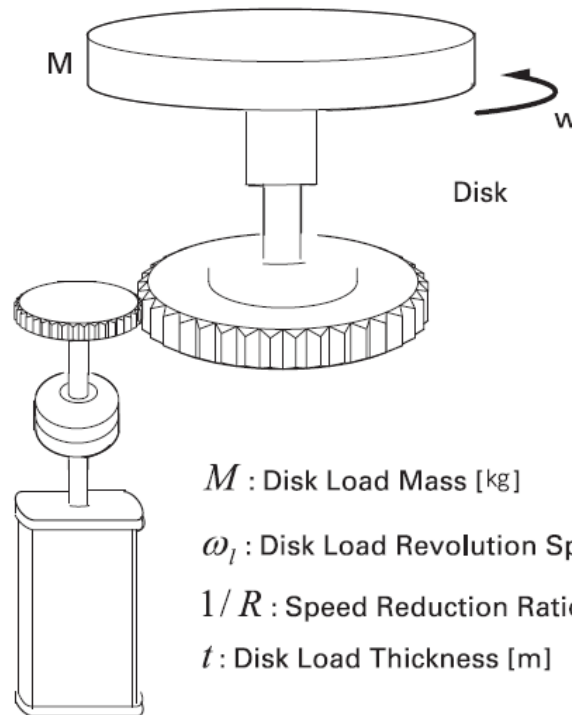
M : Disk Load Mass [kg]

ω_l : Disk Load Revolution Speed [rpm]

$1/R$: Speed Reduction Ratio

t : Disk Load Thickness [m]

T_l : Load Torque

D : Disk Load Diameter [m]

η : Mechanical Efficiency

Movement Amount (rad)

$$\theta_s = \frac{\omega_l}{60} \times \frac{2t_s - t_a - t_d}{2}$$

$$if \quad t_a = t_d, \quad \theta_s = \frac{\omega_l}{60} \times (t_s - t_a)$$

Motor Shaft Revolving Speed (r/min)

$$N_M = R\omega_l$$

Load Torque (N· m)

$$T_L = \frac{T_l}{R}$$

Load Inertia Moment (kg· m^2)

$$J_L = J_G + \frac{J_W}{R^2}$$

J_W : Disk Load Inertia, J_G : Gear, Coupling Inertia

$$J_R = \frac{MD^2}{8} = \frac{\pi \rho t D^4}{32}$$

$$\rho = 7.87 \times 10^3 \, [\text{kg/ m}^3] \, ; \text{Iron}$$

$$\rho = 2.70 \times 10^3 \, [\text{kg/ m}^3] \, ; \text{Aluminum}$$

Minimum Acceleration Time(s)

$$t_{am} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} - T_L)}$$

J_M : Motor Inertia $\quad T_{PM}$: Motor Maximum Torque

Minimum Deceleration Time(s)

$$t_{dm} = \frac{2\pi N_M (J_M + J_L)}{60(T_{PM} + T_L)}$$

Load Operation Power (W)

$$P_o = \frac{2\pi N_M T_L}{60}$$

Load Acceleration Power (W)

$$P_a = (\frac{2\pi N_M}{60})^2 \times \frac{J_L}{t_a}, \ \ (t_a \le t_{am})$$

Acceleration Torque Required (N·m)

$$T_P = \frac{2\pi N_M (J_M + J_L)}{60 t_a} + T_L, \ \ (t_a \le t_{am})$$
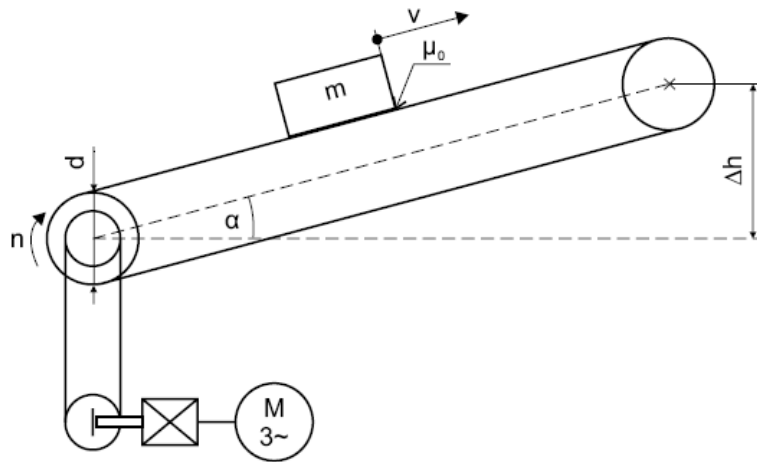
Deceleration Torque Required (N·m)

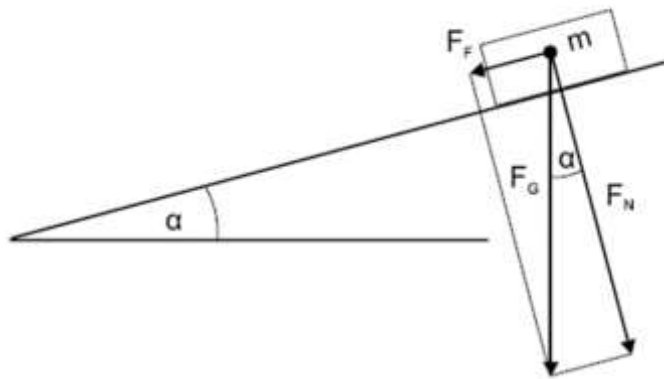$$T_S = \frac{2\pi N_M (J_M + J_L)}{60 t_d} - T_L, \ \ (t_a \le t_{dm})$$

Torque Effective Value (N·m)

$$T_{rms} = \sqrt{\frac{T_p^2 t_a + T_L^2 (t_s - t_a - t_d) + T_s^2 t_d}{t_c}}$$

$$F_N = m \cdot g \cdot \cos \alpha$$

$$F_G = m \cdot g \cdot \sin \alpha$$

$$m \cdot a_{\max} < \mu_0 \cdot m \cdot g \cdot \cos \alpha$$

$$a_{\max} < \mu_0 \cdot g \cdot \cos \alpha$$

$$F_{fr} = \mu' \cdot m \cdot g \cdot \cos \alpha$$

Fundamental Principles

1. **Forces on an Inclined Plane**:
   - The weight force F_G of the mass m is decomposed into two components due to the inclination α: the normal force F_N and the force parallel to the inclined plane F_F.
   - $F_N = F_G \cos(\alpha)$ and $F_F = F_G \sin(\alpha)$, where $F_G = m \times g$ (with g being the acceleration due to gravity).

2. **Friction Considerations**:
   - **Static Friction at the Base**: There is a static friction coefficient μ_0 between the mass and the moving base. This limits the maximum acceleration a_{max} to ensure the acceleration force F_A does not exceed the frictional force, preventing slippage.
   - **Rolling Friction on Conveyor**: When using a roller belt, the primary resistance comes from rolling friction μ', which is significantly lower than the sliding friction experienced in a traditional belt conveyor system.

Implications for Drive System Dimensioning

1. **Determining Maximum Acceleration**:
   - With $\mu_0 = 0.1$ and small α values, $a_{\max}$ must stay below 1 m/s² to avoid slippage.
   - The frictional force $F_{\text{friction}} = \mu_0 \times F_N$ acts as the upper limit for the force due to acceleration.

2. **Drive Torque and Rotation Speed**:
   - The required drive torque and rotation speed depend on the force needed to move the mass up the incline, considering both the frictional forces and the force required to overcome the inclined plane's resistance.
   - Dynamic considerations, such as starting, stopping, and speed changes, further complicate these calculations.

3. **Rolling vs. Sliding Friction**:
   - The choice between a roller belt and a traditional conveyor impacts the required drive force. Rolling friction presents less resistance, allowing for more efficient operation, particularly in systems where power conservation and efficiency are paramount.

Practical Considerations

- **System Design**: The drive system must be designed to handle the peak torque and speed requirements without exceeding the maximum allowable acceleration due to friction limitations.

- **Material Selection**: Choosing the right materials for the mass and conveyor interface to optimize friction coefficients is vital.

- **Energy Efficiency**: Systems with lower friction coefficients (like roller belts) can be more energy-efficient, reducing operational costs.

Conclusion

In summary, understanding the interplay of forces on an inclined plane and the implications of friction types is crucial in the design and dimensioning of drive systems for conveyor applications. The balance between ensuring efficient motion and preventing slippage, while optimizing for energy efficiency and system longevity, is a key aspect of mechatronic system design in dynamic applications. This knowledge forms the foundation for designing more effective and efficient mechatronic systems in various industrial applications.
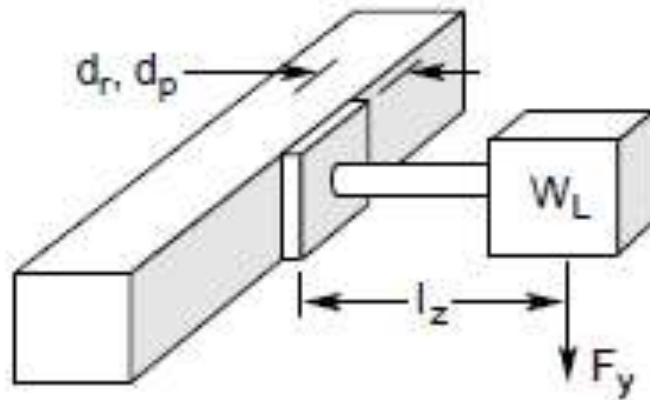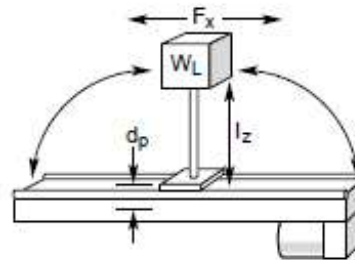
$$F_y = W_L$$
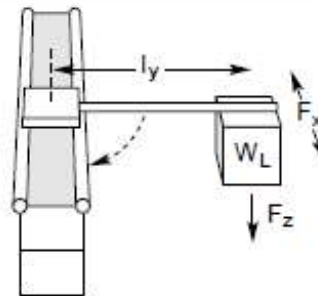$$M_p = F_x \, (l_z + d_p)$$
$$M_r = F_y \, (l_z + d_r)$$
$$F_x = \text{THRUST}$$

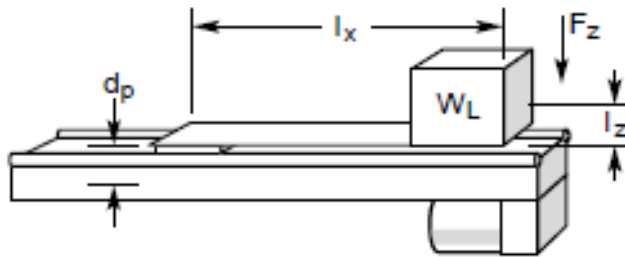$$F_n = F_z = W_L$$
$$M_p = F_x \, (l_z + d_p)$$
$$F_x = \text{THRUST}$$

$$F_n = F_z = W_L$$
$$M_p = F_x \, (l_z + d_p)$$
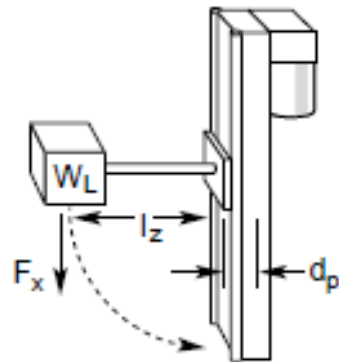$$M_r = F_z \, (l_y)$$
$$M_y = F_x \, (l_y)$$
$$F_x = \text{THRUST}$$
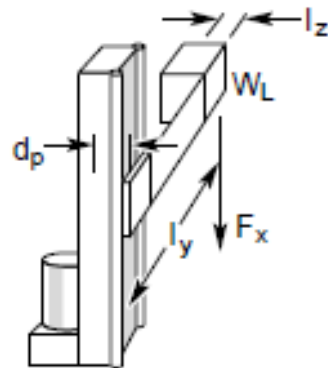
$$F_n = F_z = W_L$$
$$M_p = F_z\,(l_x) + F_x\,(l_z + d_p)$$
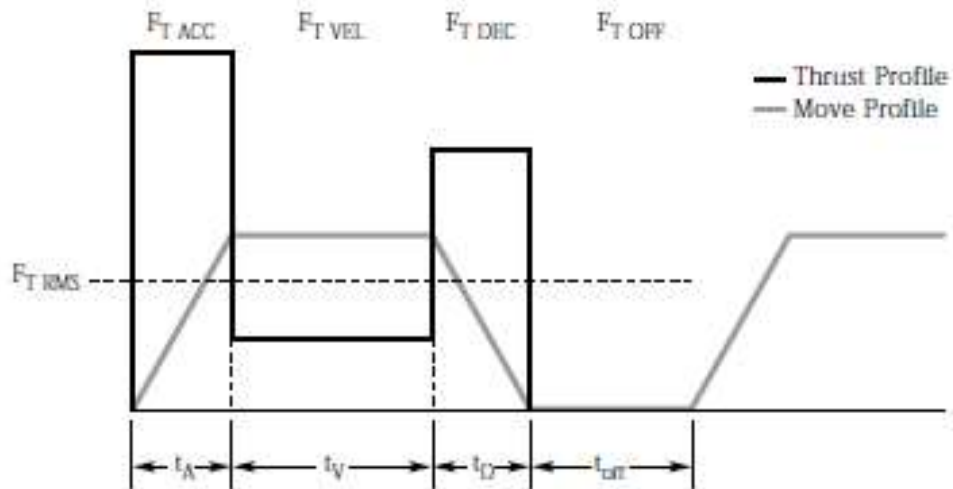$$F_x = \text{THRUST}$$

$$M_p = F_x\,(l_z + d_p)$$
$$F_x = \text{THRUST}$$

$$M_p = F_x\,(l_z + d_p)$$
$$M_y = F_x\,(l_y)$$
$$F_x = \text{THRUST}$$

$$F_{T\ RMS} = \sqrt{\dfrac{(F_{T\ ACC})^2 t_a + (F_{T\ VEL})^2 t_v + (F_{T\ DEC})^2 t_d + (F_{T\ OFF})^2 t_{off}}{t_a + t_v + t_d + t_{off}}}$$

Thrust Profile
Move Profile

$F_{T\ ACC}$ $F_{T\ VEL}$ $F_{T\ DEC}$ $F_{T\ OFF}$

$F_{T\ RMS}$

t_A t_V t_D t_{off}
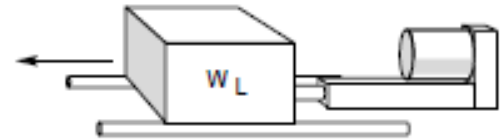
$$F_{actuator} = (W_t/g)a + F_{applied} + F_{friction}$$
$$F_{actuator} = (W_t/g)a + F_{applied} + \mu W_L$$
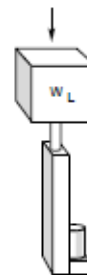
$$F_{actuator} = (W_t/g)a + F_{applied} + F_{gravity}$$
$$F_{actuator} = (W_t/g)a + F_{applied} + W_L$$

$$J_1 = m_{Belt} r_{DP}^2 + J_{IP}\left(\frac{r_{DP}}{r_{IP}}\right)^2 + \left(J_{DnP} + J_{Load}\right)\left(\frac{r_{DP}}{r_{DnP}}\right)^2$$

$$J_{DM} = J_{DP} + J_1$$

Quadrant 1: Motoring, +ve ω, +ve T

$$T_{DM} = J_{DP}\,\alpha_{DM} + \frac{1}{\eta}\left[J_1\alpha_{DM} + T_{Thrust}\left(\frac{r_{DP}}{r_{DnP}}\right)\right]$$

Quadrant 2: Braking, +ve ω, -ve T

$$T_{DM} = J_{DP}\,\alpha_{DM} + \eta\left[J_1\alpha_{DM} + T_{Thrust}\left(\frac{r_{DP}}{r_{DnP}}\right)\right]$$

Quadrant 3: Motoring, -ve ω, -ve T

$$T_{DM} = J_{DP}\,\alpha_{DM} + \frac{1}{\eta}\left[J_1\alpha_{DM} - T_{Thrust}\left(\frac{r_{DP}}{r_{DnP}}\right)\right]$$

Quadrant 4: Braking, -ve ω, +ve T
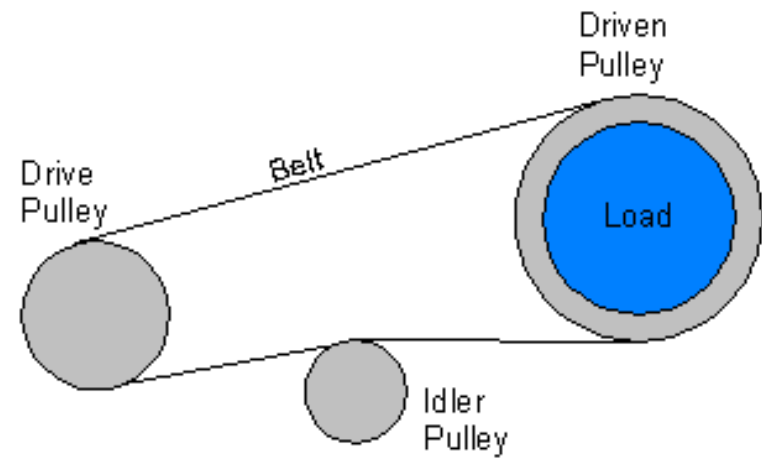
$$T_{DM} = J_{DP}\,\alpha_{DM} + \eta\left[J_1\alpha_{DM} - T_{Thrust}\left(\frac{r_{DP}}{r_{DnP}}\right)\right]$$

Symbol	Description	Units
J_{DM}	Inertia of drive mechanism	kg-m²
T_{DM}	Torque of drive mechanism	Nm
α_{DM}	Rotational acceleration	rad/s²
η	Efficiency of drive mechanism	
J_{Load}	Inertia of load + payload	kg-m²
T_{Thrust}	Thrust torque at load	N-m
J_{DP}	Inertia of drive pulley	kg-m²
J_{DnP}	Inertia of driven pulley	kg-m²
J_{IP}	Inertia of idler pulley	kg-m²
r_{DP}	Radius of drive pulley	m
r_{DnP}	Radius of driven pulley	m
r_{IP}	Radius of idler pulley	m

Driven Pulley

Load

Belt

Incline

Drive Pulley

Idler

Counterbalance Mass
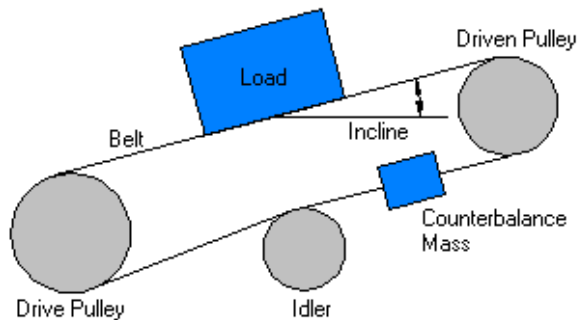
$$J_1 = \left(m_{Load} + m_{CB} + m_{Belt}\right)r_{DP}^2 + J_{IP}\left(\frac{r_{DP}}{r_{IP}}\right)^2 + J_{DnP}\left(\frac{r_{DP}}{r_{DnP}}\right)^2$$

$$J_{DM} = J_{DP} + J_1$$

Quadrant 1: Motoring, +ve ω, +ve T

$$T_{DM} = J_{DP}\,\alpha_{DM} + \frac{1}{\eta}\left[J_1\alpha_{DM} + \left(\left(m_{Load} - m_{CB}\right)g\sin\theta + F_{Thrust}\right)r_{DP}\right]$$

Quadrant 2: Braking, +ve ω, -ve T

$$T_{DM} = J_{DP}\alpha_{DM} + \eta\left[J_1\alpha_{DM} + \left(\left(m_{Load} - m_{CB}\right)g\sin\theta + F_{Thrust}\right)r_{DP}\right]$$

Quadrant 3: Motoring, -ve ω, -ve T

$$T_{DM} = J_{DP}\alpha_{DM} + \frac{1}{\eta}\left[J_1\alpha_{DM} + \left(\left(m_{Load} - m_{CB}\right)g\sin\theta - F_{Thrust}\right)r_{DP}\right]$$

Quadrant 4: Braking, -ve ω, +ve T

$$T_{DM} = J_{DP}\alpha_{DM} + \eta\left[J_1\alpha_{DM} + \left(\left(m_{Load} - m_{CB}\right)g\sin\theta - F_{Thrust}\right)r_{DP}\right]$$

Symbol	Description	Units
J_{DM}	Inertia of drive mechanism	kg-m^2
T_{DM}	Torque of drive mechanism	Nm
α_{DM}	Rotational acceleration	rad/s^2
η	Efficiency of drive mechanism	
θ	Incline angle above horizontal	o
g	Acceleration due to gravity = 9.81	m/s^2
m_{Load}	Mass of load + payload	kg
m_{CB}	Mass of counterbalance	kg
m_{Belt}	Mass of belt	kg
F_{Thrust}	Thrust force at load	N
J_{DP}	Inertia of drive pulley	kg-m^2
J_{DnP}	Inertia of driven pulley	kg-m^2
J_{IP}	Inertia of idler pulley	kg-m^2
r_{DP}	Radius of drive pulley	m
r_{DnP}	Radius of driven pulley	m
r_{IP}	Radius of idler pulley	m
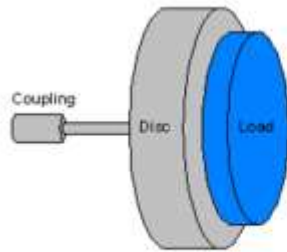
Coupling

Disc

Load

Unbalanced Load
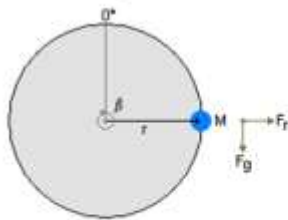
Front View
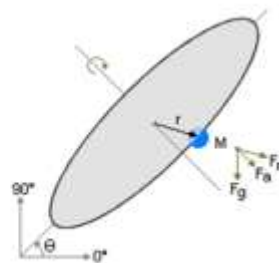
Incline Above Horizontal

$$J_1 = J_{Disc} + J_{Load} + Mr^2$$
$$J_{DM} = J_{Coupling} + J_1$$

Quadrant 1: Motoring, +ve ω, +ve T

$$T_{DM} = J_{Coupling}\alpha_{DM} + \frac{1}{\eta}\left[J_1\alpha_{DM} + T_{Thrust} + Mgrsin(\phi + \beta)sin(\theta)\right]$$

Quadrant 2: Braking, +ve ω, -ve T

$$T_{DM} = J_{Coupling}\alpha_{DM} + \eta\left[J_1\alpha_{DM} + T_{Thrust} + Mgrsin(\phi + \beta)sin(\theta)\right]$$

Quadrant 3: Motoring, -ve ω, -ve T

$$T_{DM} = J_{Coupling}\alpha_{DM} + \frac{1}{\eta}\left[J_1\alpha_{DM} - T_{Thrust} + Mgrsin(\phi + \beta)sin(\theta)\right]$$
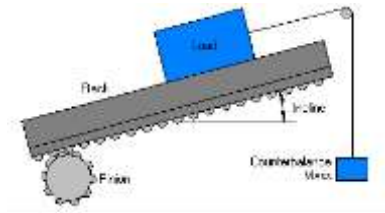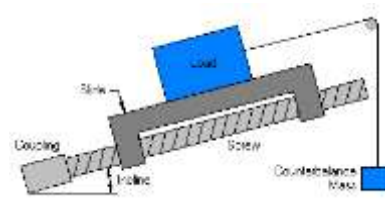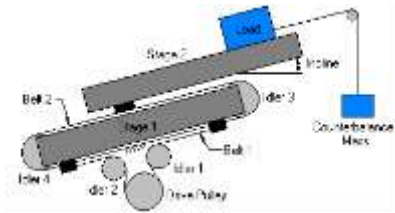
Quadrant 4: Braking, -ve ω, +ve T

$$T_{DM} = J_{Coupling}\alpha_{DM} + \eta\left[J_1\alpha_{DM} - T_{Thrust} + Mgrsin(\phi + \beta)sin(\theta)\right]$$

$$J_1 = n_{PR} J_{PR} \left(\frac{r_{DR}}{r_{PR}} \right)^2 + m_{Loop} r_{DR}^2$$

$$J_{DM} = n_{DR} J_{DR} + J_1$$

Quadrant 1: Motoring, +ve ω, +ve T

$$T_{DM} = n_{DR} J_{DR} \alpha_{DM} + \frac{1}{\eta} \left[J_1 \alpha_{DM} + \left(m_{Loop} g + F_{Thrust} \right) r_{DR} \right]$$

Quadrant 2: Braking, +ve ω, -ve T

$$T_{DM} = n_{DR} J_{DR} \alpha_{DM} + \eta \left[J_1 \alpha_{DM} + \left(m_{Loop} g + F_{Thrust} \right) r_{DR} \right]$$

Quadrant 3: Motoring, -ve ω, -ve T

$$T_{DM} = n_{DR} J_{DR} \alpha_{DM} + \frac{1}{\eta} \left[J_1 \alpha_{DM} + \left(m_{Loop} g - F_{Thrust} \right) r_{DR} \right]$$

Quadrant 4: Braking, -ve ω, +ve T

$$T_{DM} = n_{DR} J_{DR} \alpha_{DM} + \eta \left[J_1 \alpha_{DM} + \left(m_{Loop} g - F_{Thrust} \right) r_{DR} \right]$$

Pinch Rolls

Loop

Drive Rolls

Left diagram:

Connecting Rod

Crank

Load

Offset

Incline

Position

− +

Right diagram:

Load

Table

Secondary

Primary

Incline

Counterbalance Mass

(a) Flywheel $I = MR^2$ (b) Solid wheel $I = \frac{1}{2}MR^2$ (c) Axis at end $I = \frac{1}{3}ML^2$ (d) Axis in middle $I = \frac{1}{12}ML^2$
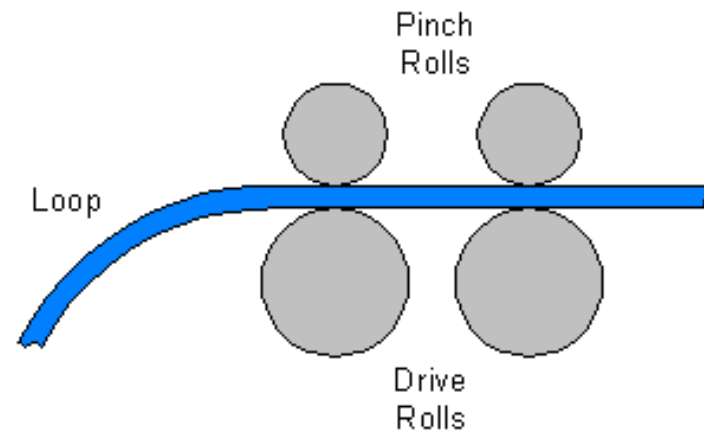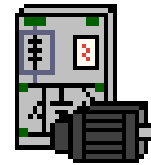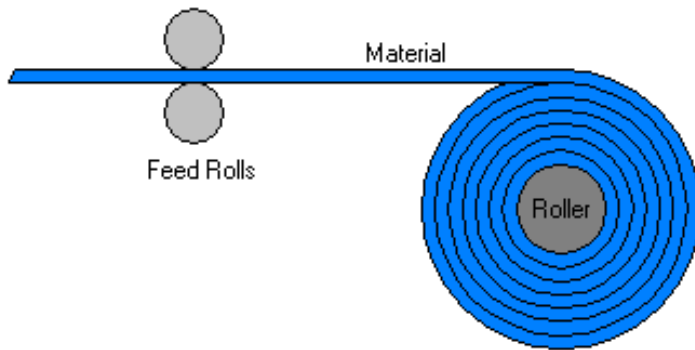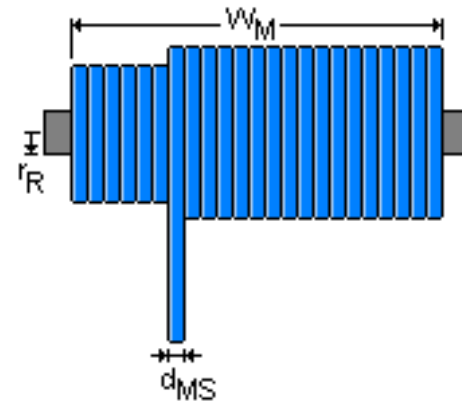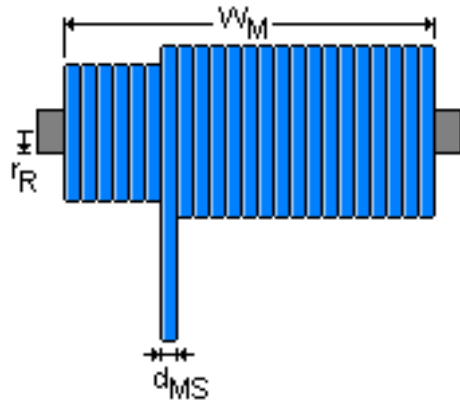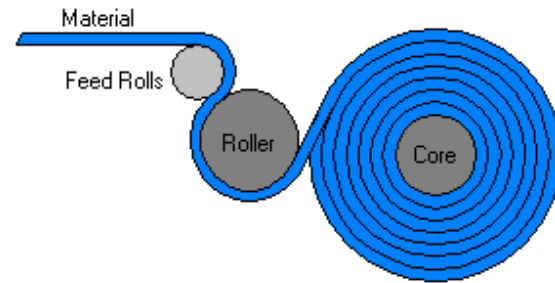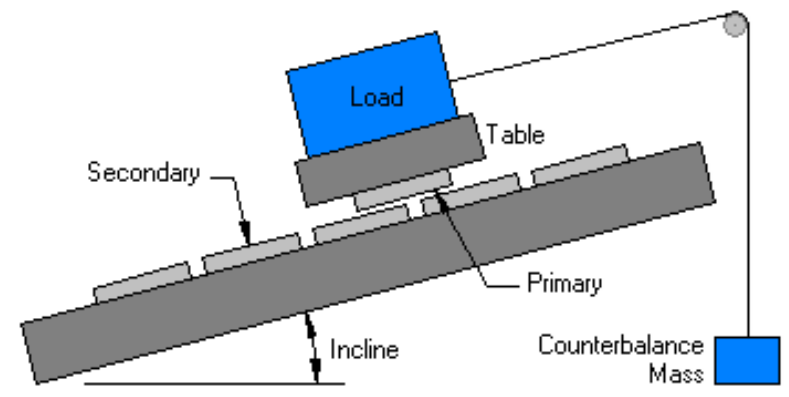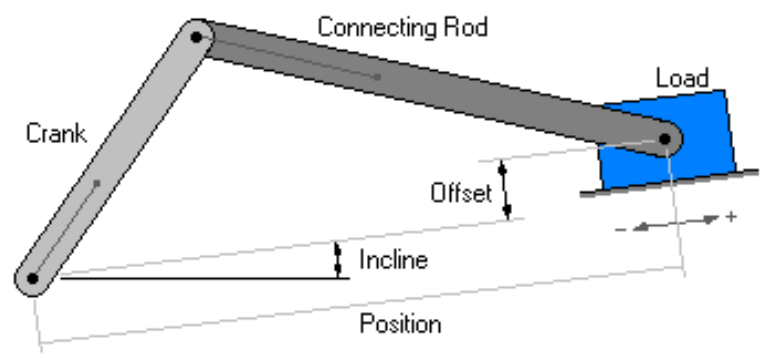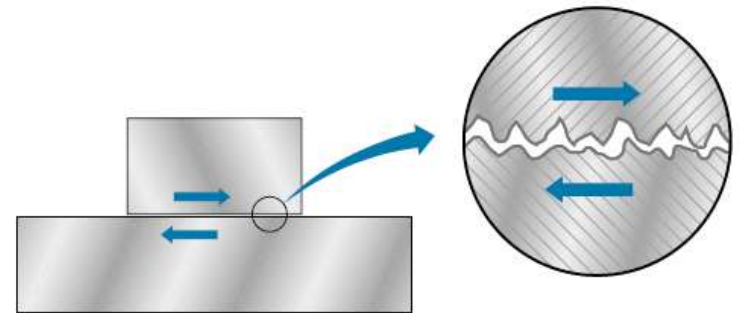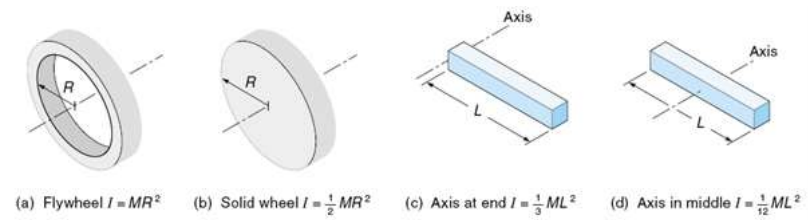
Behavior of Mechanical Components
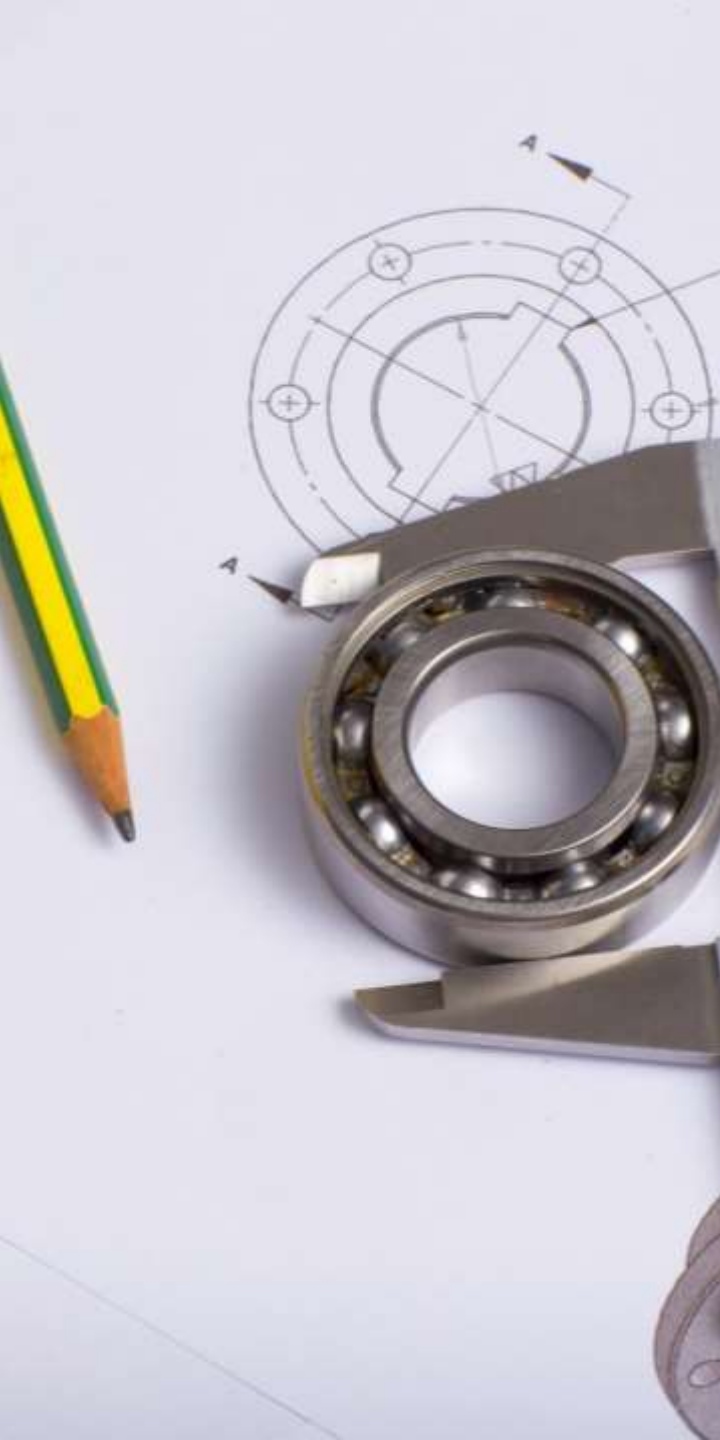
Understanding Friction

- **Friction Dynamics**: Friction, particularly sliding friction, occurs when parts slide against each other. It's typically caused by surface irregularities, often referred to as high spots, that resist motion.

- **Friction Force**: This force is directly proportional to the normal force (N), which is the force pressing the parts together. Normal force could be due to the weight of the moving part or externally applied forces like a bolt or spring.

- **Dilemma in Design**: Achieving a balance between snug fit and frictional resistance is challenging. A snug fit reduces rattles but increases friction.

- **Static Friction**: It is crucial to note that static friction (friction at startup) is typically higher than dynamic friction (friction during motion), leading to a common issue where parts tend to overshoot their intended position once static friction is overcome.

- **Coefficient of Friction**: This is a material-dependent value, used to calculate the force needed to overcome friction. However, these values can vary and should be considered approximations.

Lubrication and Its Impact

Role of Lubricants: Proper lubrication can significantly reduce friction, ideally preventing direct contact between parts. The resistance in this case depends on the relative velocity of the moving parts, contrasting with the near-constant drag force in sliding friction.

Characteristics of Good Lubricants: Effective lubricants must adhere to surfaces without impeding motion. However, contaminants like dirt and sand can adhere to the lubricant, increasing friction and potentially damaging parts.

Dry Lubricants and Bearing Design: Certain applications use dry lubricants for reduced friction without stickiness. It's essential to follow manual guidelines for lubrication, as some joints are designed to operate without it.

Rolling Friction and Mechanical Efficiency

Advantages of Rolling Friction: It offers considerably less resistance compared to sliding friction. This principle is employed in ball bearings and roller bearings to achieve low friction values.

Energy Conversion and Efficiency: The energy used to overcome friction is often converted to heat. The efficiency of a mechanical energy transport system (like bearings or gearboxes) is a measure of the percentage of input mechanical power that is effectively used, with the remainder lost to friction.

Coefficient of Friction for Various Materials*

Material	Static	Sliding
Steel on steel	0.78	0.42
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.40
Oak on oak	0.62	0.48
Rubber on pavement	0.9	0.85
Diamond on diamond	0.2	
Teflon on Teflon	0.04	

*from Marks *Standard Handbook for Mechanical Engineers* 1967 and others

Friction

Friction occurs in any mechanical system. It always acts against the movement of the body. If a body is to be moved along a path, friction forces will arise. These may be differentiated according to their causes.

The most important types of straight-line movement are static friction, sliding friction, and rolling friction. The friction force F_{fr} *is proportional to* the normal force F_N, *by which the b body is pressed against the friction surface.*

This relationship is described by the coefficient of friction μ. $$F_{fr} = \mu \cdot F_N$$
In all horizontal movements, the normal force is equal to the force due to weight. In these cases, the friction force is calculated following the equation:

$$F_{fr} = \mu \cdot g \cdot m$$

Static friction occurs when a force is applied to a movable body all the time this body remains in a resting position. Once the body begins to move, static friction is reduced and sliding friction occurs. Static friction and sliding friction are also known as Coulomb's friction. The coefficient of friction depends on the material pairing and any lubricant present (water, oil). Typical coefficients of static friction μ0 lie in the range from 0.15 to 0.8, while the coefficients of sliding friction μ lie between 0.1 and 0.6.
The principle of rolling is frequently used in technology. Rolling friction is significantly less than sliding friction. Typical coefficients of rolling friction μ' lie in the range of 0.002 to 0.04.

Friction plays an important role in movement processes. Thus a proportion of the power loss within the drive components is due to friction forces.

It is therefore important to use components that are as low loss as possible.

In the case of horizontal movement with low dynamics, the stationary power of the drive system is generally determined by friction.

In some applications, the transfer of forces is based on the principle of friction (friction wheels, synchronized drives with rollers, cables on driving pulleys, vehicles on rails). Here the coefficient of static friction limits the maximum transferable forces and hence the maximum possible acceleration.

Friction may, however, be ignored in many dynamic applications with high accelerations or in hoist applications since the biggest part of the drive power is used for building up kinetic or potential energy.

Mechanical Vibration in Drive Trains

Vibration Sources: The vibration level in a mechanical drive train (involving components such as the motor, coupling, and load) results from cyclic forces. These forces can be due to residual imbalance in the rotor or other cyclic forces, and the system's response to these forces.

Vibration Categories: Problems typically fall into three categories:

- **High-Power, High-Speed Applications**: These require operation above the first critical speed, where resonance can be a significant issue.

- **Torque Ripple Excitations**: Applications where torque ripple induces resonance in the mechanical system.

- **High-Performance Closed-Loop Systems**: In these systems, rapid changes in motor torque can cause significant shaft twisting, with the control loop potentially sustaining the vibration.

Critical Speed and Torsional Dynamics

- **Understanding Critical Speed**: Critical speed refers to the speed at which a system's natural frequency coincides with the frequency of an external force or the system's operational speed, leading to resonance.

- **Torsional Dynamics**: Torsional dynamics involve the study of how torques and rotational movements propagate through a system. In mechanical drive trains, this often concerns the twisting of shafts linking different components and how this affects the overall system behavior.

Harmonic Torques and Variable-Speed Drives

- **Torque Ripple**: Modern variable-speed drives produce smaller torque ripples compared to older technologies. However, harmonic torques are still generated, which can excite mechanical resonances.

- **Misconceptions**: While it might seem logical to attribute system vibrations to torque ripple in electrical variable-speed drives, in practice, this is rarely the primary cause.

Systemic View of Vibrations

- **Systemic Vibrations**: Any system where masses (or inertia) are coupled through flexible elements is capable of vibrating. Even a simple setup of a motor and load can be viewed as a two-mass torsional system.

- **General Case**: In a theoretical framework with two inertias and zero damping, the equations of motion can describe the dynamic behavior of the system, including its potential for vibration and resonance.

The notch filter is used when the resonant frequency of the motor and load system is amplified by the servo system.

A simple model of the motor inertia and load inertia which are connected by a shaft with a stiffness is shown.

The center frequency of the notch filter can be programmed to cancel or minimize the gain at the resonant frequency.
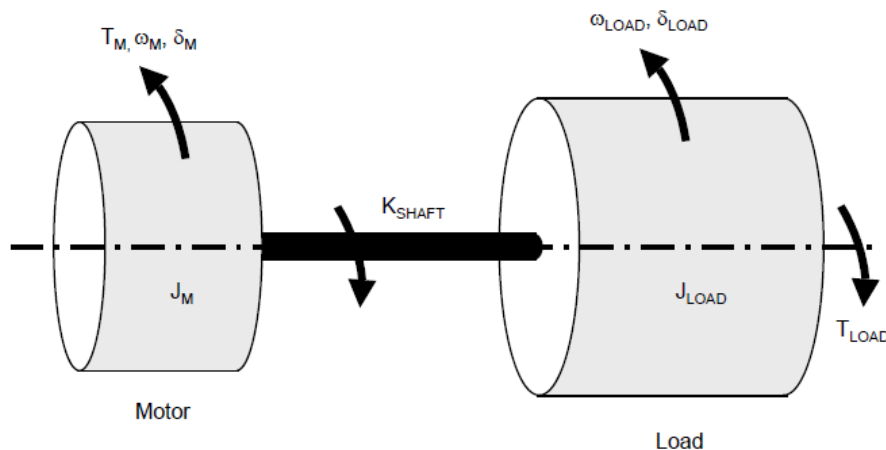
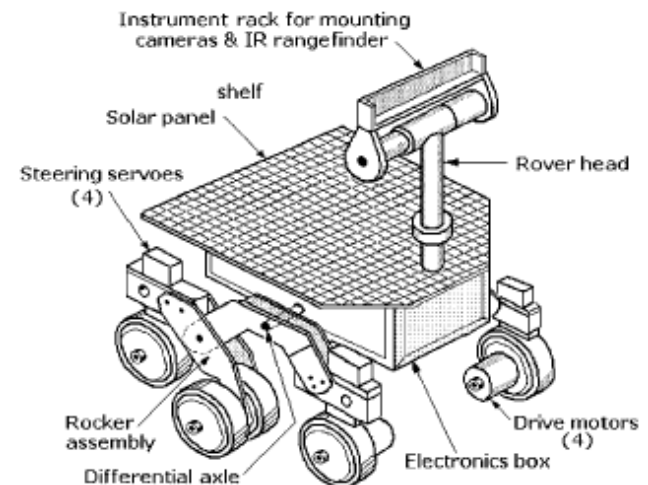Resonant Frequency $= 2\pi f = \sqrt{K_{SHAFT}\,((1/J_M)+(1/J_{LOAD}))}$ where:

J is inertia (lb-in-s^2)

K_{SHAFT} is stiffness (lb-in/rad) $= (\pi D^4 G) / (32 L)$ where:

G $= 11 \times 10^6$ lb/in^2 for stainless steel, D is shaft diameter (in), L is shaft length (in)

$$J_1(d^2\theta_1/dt^2) + K(\theta_1 - \theta_2) = 0$$

$$J_2(d^2\theta_2/dt^2) + K(\theta_2 - \theta_1) = 0$$

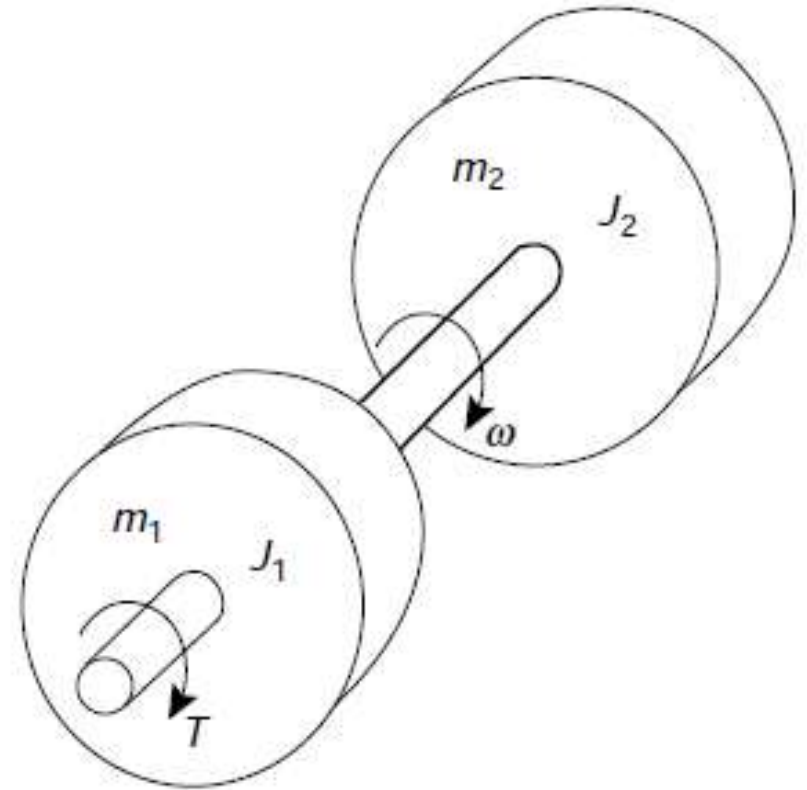Eliminating θ_2 or θ_1 gives the general solution

$$\theta = A + Bt + C\cos(\omega_{ntf} + \phi)$$

where

$$\omega_{ntf} = \sqrt{[K(J_1 + J_2)/(J_1 \times J_2)]}$$

or

$$f_{ntf} = (1/2\pi)\sqrt{[K(J_1 + J_2)/(J_1 \times J_2)]}$$

where ω_{ntf} is torsional natural frequency (rad^{-1}), f_{ntf} is torsional natural frequency (Hz) and K is torsional stiffness ($= GJ_p/L$, where G is shear modulus of elasticity, J_p is polar moment of inertia of the shaft, which is equal to $r^4/2$ for a circular shaft of radius r, and L is the length of shaft being twisted).

Example
An a.c. motor of inertia 0.5 kg m^2 is coupled to a load of inertia (0.4 kg m^2) via a shaft with a torsional spring constant of 60×10^3 N m rad^{-1}.

$$f_{ntf} = (1/2\pi)\sqrt{[k_t(J_1 + J_2)/(J_1 \times J_2)]}$$
$$= (1/2\pi)\sqrt{[60 \times 10^3(0.5 + 0.4)/(0.5 \times 0.4)]}$$
$$= 82.7 \, \text{Hz}$$

Dynamic Loading and Gearbox Sizing in Servomotor Systems

Automation and motion control applications often involve frequent starts/stops and direction changes, presenting complex load cycles.

Correct gearbox sizing is essential for servomotors, considering load inertia, maximum non-dynamic load, and motor data.

The maximum dynamic peak torque is known, but assuming the gearbox experiences this maximum can lead to oversizing.

A significant portion of the motor peak torque is used to accelerate the motor rotor, so only a part of this torque is transmitted through the gearbox.

Torque Calculation:

Equation Derivation: By considering motor peak torque (TM), rotor inertia (JM), and angular acceleration (a), we derive equations to calculate the torque through the gearbox (TGR).

Inertia Parameter (k): k = JM / (JM + JLR), where JLR is the load inertia reflected to the motor axis, helps in determining TGR.

System Inertia Matching

Inertia Match Ratio (IMR): IMR, or Load Inertia / Motor Inertia, is a critical factor in system balance. A balanced system (IMR ≈ 1:1) exhibits good control characteristics.

Application: For a system with matched inertia, only a portion of the motor peak torque is required for acceleration, impacting gearbox selection.
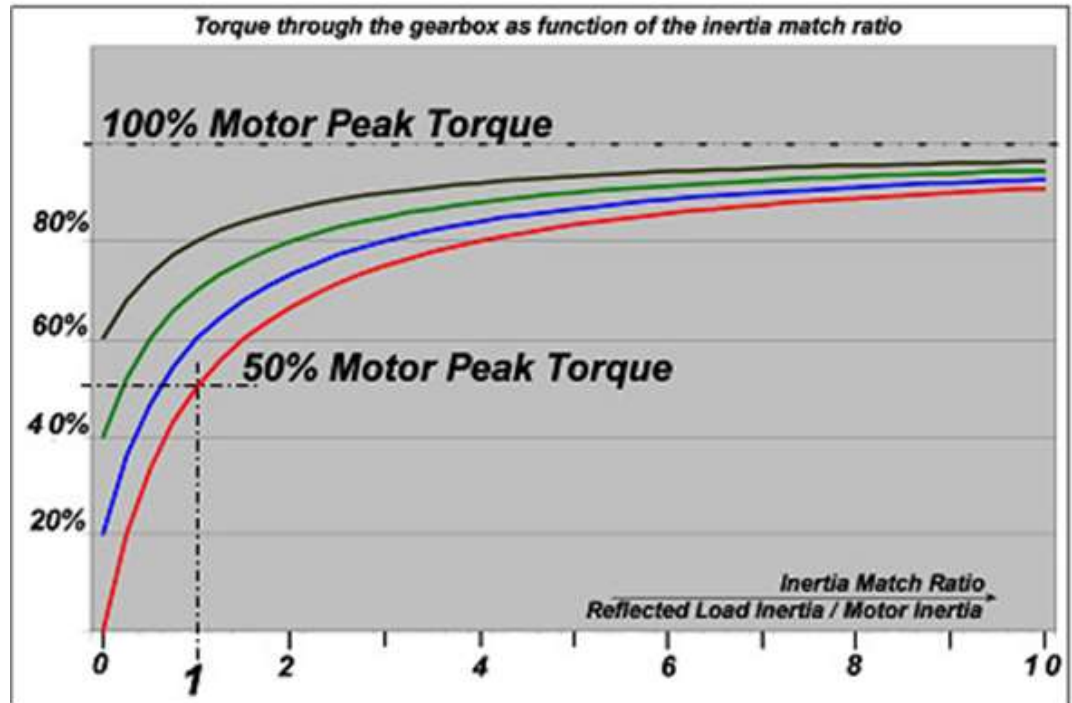

Idealized Drive System, consisting of Motor, Grearbox and Load (All parameters are reflected to the motor axis)
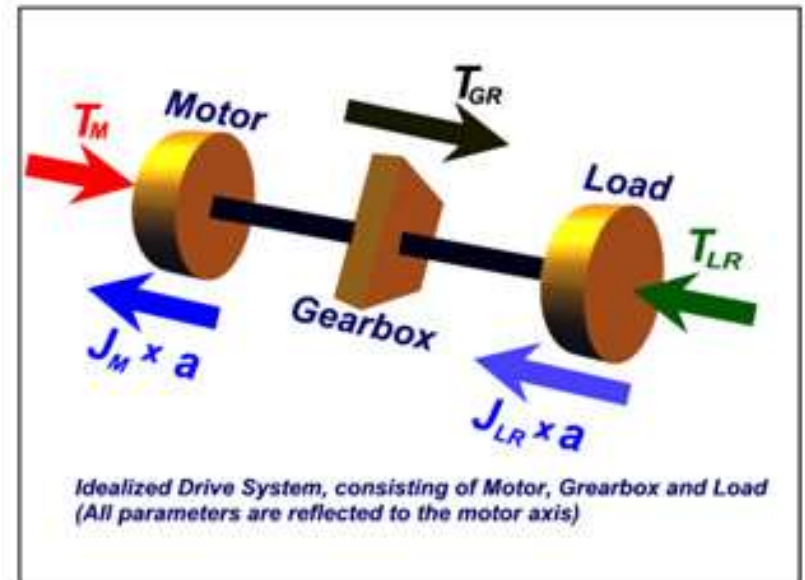
High Cycle Rate Applications:

Torque Utilization: In high cycle rate applications, full motor peak torque cannot be continuously utilized without risking motor damage.

IMR Considerations: A low IMR (close to or less than 1) is advisable for high cycle rate applications, impacting gearbox torque requirements.

Torque through the gearbox as function of the inertia match ratio

100% Motor Peak Torque

50% Motor Peak Torque

Inertia Match Ratio
Reflected Load Inertia / Motor Inertia

—— Pure dynamic load, NO friction torque load
—— Friction torque load is 20% of motor peak torque
—— Friction torque load is 40% of motor peak torque
—— Friction torque load is 60% of motor peak torque