I. GYROTROPIC EQUATION OF MOTION

In a gyroelectric medium, the polarization vector $\mathbf{P}_n$ is governed by the Landau-Lifshitz-Gilbert equation,

$$\frac{d\mathbf{P}_n}{dt} = -\mathbf{P}_n \times (\sigma \mathbf{E} + \omega_n \mathbf{b}_n) + \gamma \mathbf{P}_n \times \frac{d\mathbf{P}_n}{dt}$$

Although this is different from the damped harmonic oscillator equation used for Drude-Lorentz susceptibility, the constants σ, ω_n, and γ_n play analogous roles: σ couples the polarization to the electric field, ω_n is an angular frequency of precession, and γ_n is a damping factor. The "bias vector" b_n describes the direction of an applied static magnetic field, and is assumed to have unit length. Note that this equation of motion is time-reversal asymmetric; in the absence of an $\mathbf{E}$ field, the polarization vector executes a damped counterclockwise precession around the bias vector $\mathbf{b}_n$. The norm of the polarization vector, $P_{ns} = |\mathbf{P}_n|$, is a constant of motion.

In the frequency domain, the Landau-Lifshitz-Gilbert equation generates an ϵ tensor with skew-symmetric off-diagonal components. To see this, take $\mathbf{b}_n = \hat{\mathbf{z}}$ and decompose the polarization into a static and oscillating part, $\mathbf{P}_n = P_{ns}\mathbf{b} + \mathbf{p}_n$. Assuming both $\mathbf{p}_n$ and $\mathbf{E}$ have harmonic time-dependence $\exp(-i\omega t)$, the oscillating part gives

$$-i\omega \mathbf{p}_n = \mathbf{b} \times \Big(- \sigma P_{ns} \mathbf{E} + \omega_n \mathbf{p}_n - i\omega\gamma P_{ns} \mathbf{p}_n \Big)$$

This implies that

$$\mathbf{p}_n = \chi_n \mathbf{E}, \quad \chi_n = \frac{\sigma_n P_{ns}}{(\omega_n - i\omega\gamma_n P_{ns})^2 - \omega^2} \begin{bmatrix} \omega_n - i\omega\gamma_n P_{ns} & -i\omega & 0 \\ i\omega & \omega_n - i\omega\gamma_n P_{ns} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, the frequency domain dielectric function is

$$\epsilon = \begin{bmatrix} \epsilon_\perp & -i\eta & 0 \\ i\eta & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_\infty \end{bmatrix}, \quad \text{where} \quad \begin{cases} \epsilon_\perp & = \epsilon_\infty + \dfrac{\Omega_n(\omega_n - i\omega\alpha_n)}{(\omega_n - i\omega\alpha_n)^2 - \omega^2} \\ \eta & = \dfrac{\Omega_n \omega}{(\omega_n - i\omega\alpha_n)^2 - \omega^2} \\ \Omega_n & = \sigma_n P_{ns}. \;\; \alpha_n = \gamma_n P_{ns} \end{cases}$$

II. IMPLEMENTATION

We track $\mathbf{p}_n$ where

$$\mathbf{P}_n = P_{ns}\mathbf{b}_n + \mathbf{p}_n \tag{1}$$

The Landau-Lifshitz-Gilbert equation can be re-written as

$$\frac{d\mathbf{p}_n}{dt} = -\sigma(P_{ns}\mathbf{b}_n + \mathbf{p}_n) \times \mathbf{E} - \omega_n \mathbf{p}_n \times \mathbf{b}_n + \gamma\left(P_{ns}\mathbf{b}_n + \mathbf{p}_n\right) \times \frac{d\mathbf{p}_n}{dt} \tag{2}$$

In component terms,

$$\left[\delta_{ij} + \gamma\varepsilon_{ijk}\left(P_{ns}b_n^k + p_n^k\right)\right]\frac{dp_n^j}{dt} = \varepsilon_{ijk}\left[\sigma E^j\left(P_{ns}b_n^k + p_n^k\right) + \omega_n b_n^j\, p_n^k\right] \tag{3}$$

We can discretize this into time steps τ using standard midpoint rules:

$$\begin{aligned}\left[\delta_{ij} + \gamma\varepsilon_{ijk}\left(P_{ns}b_n^k + p_n^{k(t)}\right)\right]p_n^{j(t+1)} = &\left[\delta_{ij} + \gamma\varepsilon_{ijk}\left(P_{ns}b_n^k + p_n^{k(t)}\right)\right]p_n^{j(t-1)} \\ &+ \varepsilon_{ijk}\left[2\tau\sigma E^j\left(P_{ns}b_n^k + p_n^{k(t)}\right) + 2\tau\omega_n b_n^j\, p_n^{k(t)}\right]\end{aligned}$$

We can do the matrix inversion using the fact that

$$\begin{bmatrix} A & Z & -Y \\ -Z & A & X \\ Y & -X & A \end{bmatrix}^{-1} = \frac{1/A}{A^2+X^2+Y^2+Z^2}\begin{bmatrix} A^2+X^2 & XY-AZ & XZ+AY \\ YX+AZ & A^2+Y^2 & YZ-AX \\ ZX-AY & ZY+AX & A^2+Z^2 \end{bmatrix}. \tag{4}$$