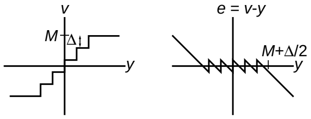
Oversampling Delta-Sigma Data Converters
The One-Page Story

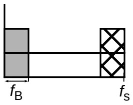
Quantization



As long as the quantizer does not overload $|e| \le \Delta/2$.

If i) the quantizer does not overload, ii) the input to the quantizer is busy and iii) the number of quantization levels is large, then the quantization noise is white with a power $\sigma_e^2 = \Delta^2/12 = 1/3$ for $\Delta=2$.

Oversampling

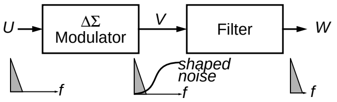


$$OSR = \frac{f_s}{2 f_B}$$

For white noise, the power in the band-of-interest is the power of the signal divided by *OSR*.
⇒ Oversampling reduces noise.

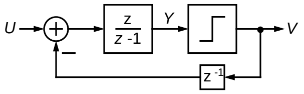
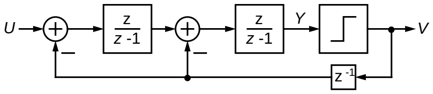
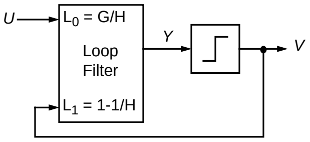
The first alias is approximately 2*OSR* times higher in frequency than the upper passband edge.
⇒ Anti-aliasing requirements are relaxed by oversampling.

Basic ΔΣ Architecture



For an ADC system, the modulator is analog and the (decimation) filter is digital. The anti-alias filter which precedes the modulator is not shown.

For a DAC system, the modulator is digital and the filter is analog. The interpolation filter which precedes the modulator is not shown.

MOD1	MOD2	MODn
 $V(z) = U(z) + (1 - z^{-1})E(z)$	 $V(z) = U(z) + (1 - z^{-1})^2 E(z)$	 $V(z) = G(z)U(z) + H(z)E(z)$

`double u, v=0, x=0;`
`while( cin >> u ){`
`     x += u - v;`
`     v = x<0?-1:1;`
`     cout << v << endl;`
`     }`
	`double u, v=0, x1=0, x2=0;`
	`while( cin >> u ){`
	`     x1 +=  u - v;`
	`     x2 += x1 - v;`
	`     v = x2<0?-1:1;`
	`     cout << v << endl;`
	`     }`
		`H = synthesizeNTF(n=3,OSR=64, ...`
		`     opt=0,Hinf=1.5,f0=0);`
		
		`[v xn xmax y] = simulateDSM(u,H,nlev=2,x0=0);`

$IQNP = \frac{\pi^2\sigma_e^2}{3(OSR)^3} \begin{cases} = 2\sigma_e^2(OSR)^{-2} & \text{for sinc}^1 \\ = 2\sigma_e^2(OSR)^{-3} & \text{for sinc}^2 \end{cases}$	$IQNP = \frac{\pi^4\sigma_e^2}{5(OSR)^5} \begin{cases} = 6\sigma_e^2(OSR)^{-4} & \text{for sinc}^2 \\ = 6\sigma_e^2(OSR)^{-5} & \text{for sinc}^3 \end{cases}$	$IQNP = \frac{\sigma_H^2\sigma_e^2}{OSR}$ `sigma_H = rmsGain(H,0,0.5/OSR);`
$(\lvert u\rvert \le 1) \Rightarrow (\lvert x\rvert \le 2)$	$(u \text{ constant}, \lvert u\rvert \le 1) \Rightarrow$ $\left(\lvert x_1\rvert \le \lvert u\rvert + 2, \quad \lvert x_2\rvert \le \frac{(5 - \lvert u\rvert)^2}{8(1 - \lvert u\rvert)}\right)$	Lee’s Rule (with margin): $\lVert H\rVert_\infty < 1.5 \Rightarrow$ probably stable
Quantization noise is not white– susceptible to “limit cycles,” esp. with small DC inputs. For DC inputs, the output spectrum is discrete.	Less susceptible to limit cycles. Frequent quantizer overload (which leads to excess quantization noise) can be avoided by the use of a less aggressive NTF.	Can trade-off stability (in terms of the frequency of quantizer overload or in terms of the maximum stable input) for increased noise suppression by increasing the bound on $\lVert H\rVert_\infty$.

ΔΣ Toolbox Quick Reference

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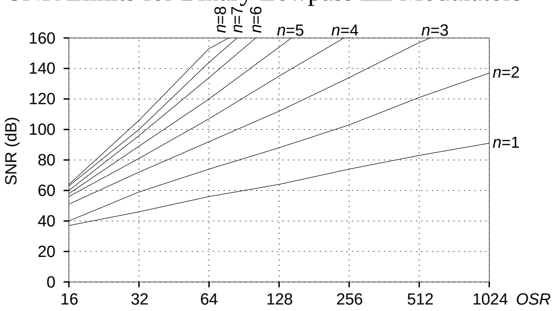
MAIN FUNCTIONS
ntf = synthesizeNTF(order=3,OSR=64,opt=0,H_inf=1.5,f0=0)
ntf = clans(order=4,OSR=64,Q=5,rmax=0.95,opt=0)
[snr,amp,k0,k1,sigma_e2] = predictSNR(ntf,OSR=64,amp=...,f0=0)
[v,xn,xmax,y] = simulateDSM(u,ABCD,nlev=2,x0=0) or
[v,xn,xmax,y] = simulateDSM(u,ntf,nlev=2,x0=0)
[snr,amp] = simulateSNR(ntf,OSR,amp=...,f0=0,nlev=2,f=1/(4*OSR),k=13)
[a,g,b,c] = realizeNTF(ntf,form='CRFB',stf=1)
ABCD = stuffABCD(a,g,b,c,form='CRFB')
[a,g,b,c] = mapABCD(ABCD,form='CRFB')
[ABCDs,umax]=scaleABCD(ABCD,nlev=2,f=0,xlim=1,ymax=nlev+5,umax,N=1e5)
[ntf,stf] = calculateTF(ABCD,k=1)
[gu,gv,H,L0,L0k] = designLCBP(n=3,f0=1/16,fb=1/128,Hinf=1.6,t1=0,…)
[sv,sx,sigma_se,max_sx,max_sy] = simulateESL(v,mtf,M=16,dw=[1…],sx0=[0…])
[f1,f2,info] = designHBF(fp=0.2,delta=1e-5,debug=0)
[s,e,n,o,Sc] = findPIS(u,ABCD,nlev=2,options)

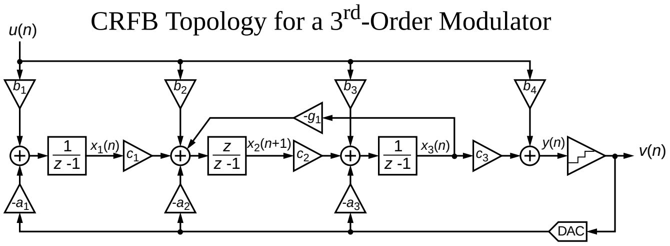
AUXILIARY FUNCTIONS
window = ds_hann(N)
snr = calculateSNR(hwfft,f)
sigma_H = rmsGain(H,f1,f2)
H_inf = infnorm(H)
[A B C D] = partitionABCD(ABCD, m)
tf_z = evalTF(tf,z)
figureMagic( xRange, dx, xLab, yRange, dy, yLab, size )
printmif(file,size,font,fig)

The toolbox is available from http://www.mathworks.com/matlabcentral/
fileexchange.
```

SNR Limits for Binary Lowpass ΔΣ Modulators



CRFB Topology for a 3rd-Order Modulator



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