Alpha-Beta filter

February 16, 2021

Correction equations:

$$\hat{x}_k \leftarrow \hat{x}_k + \alpha \hat{r}_k \tag{1}$$

$$\hat{v}_k \leftarrow \hat{v}_k + \frac{\beta}{\Delta T}\hat{r}_k, \tag{2}$$

with

$$0 < \alpha^2 < 1 \tag{3}$$

$$0 < \beta \leq 2 \tag{4}$$

$$0 < 4 - 2\alpha - \beta. \tag{5}$$

In the code we have

$$\hat{x}_k \leftarrow \hat{x}_k + \alpha \hat{r}_k \Delta T \tag{6}$$

$$\hat{v}_k \leftarrow \hat{v}_k + \alpha^2 \hat{r}_k \Delta T. \tag{7}$$

For these two system to be equal we must have

$$\frac{\alpha \hat{r}_k}{\frac{\beta}{\Delta T}\hat{r}_k} = \frac{\alpha \hat{r}_k \Delta T}{\alpha^2 \hat{r}_k \Delta T} \implies \alpha^3 \Delta T = \alpha \frac{\beta}{\Delta T}\Delta T = \alpha\beta \implies \beta = \alpha^2 \Delta T.$$

With defaults $\alpha = 0.35$ and a 2kHz loop frequency this means $\beta = 0.35^2 \times \frac{1}{2000} = 6.125 \times 10^{-5}$.