A real-time digital twin of nonlinear azimuthal thermoacoustics

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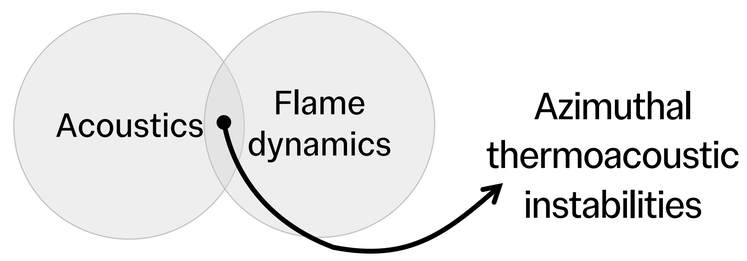
Context & objective

- A digital twin is a real-time model combining measured data with physical predictions to mimic its physical counterpart.
- We create a digital twin of azimuthal thermoacoustic oscillations, crucial for hydrogen-based and sustainable aviation fuels.
- Thermoacoustics are a nonlinear multi-physics phenomenon, which can compromise aeroengine safety.
- We model two of the key subsystems and their interaction: the acoustics and flame dynamics.



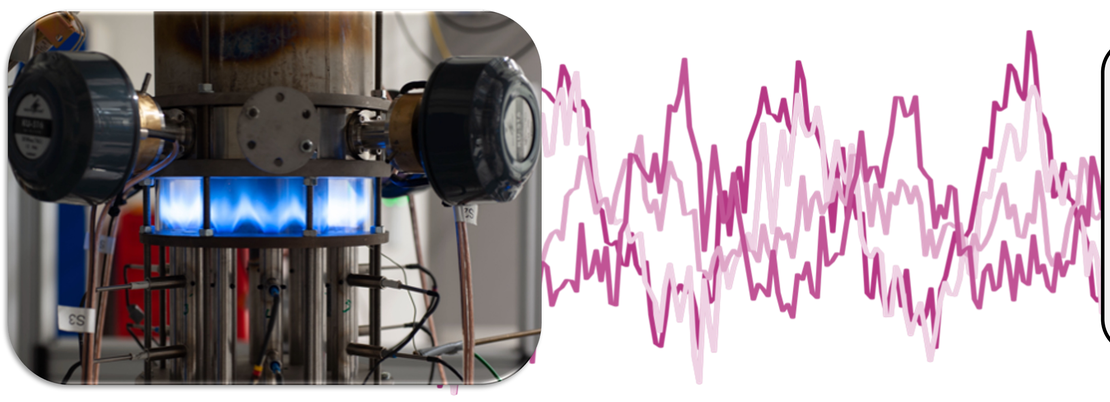
Components of the real-time digital twin

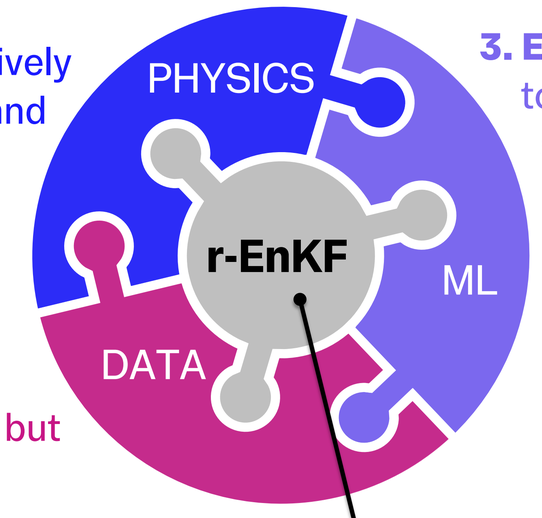
1. Low-order model: qualitatively accurate, but quantitatively inaccurate, with aleatoric uncertainties in the states $\boldsymbol{\phi}$ and parameters $\boldsymbol{\alpha}$, as well as biases (model errors).

$$\begin{cases} \dfrac{\partial^2 p}{\partial t^2} + \zeta \dfrac{\partial p}{\partial t} - \omega^2 \dfrac{\partial^2 p}{\partial \theta^2} = \dfrac{\partial \dot{Q}}{\partial t} \\ \dot{Q} = \beta\left[1 + c_2 \cos 2(\theta - \Theta_\beta)\right] p - \kappa p^3 \end{cases} \Rightarrow \begin{array}{c} \mathrm{d}\boldsymbol{\psi} = \boldsymbol{F}(\boldsymbol{\phi}, \boldsymbol{\alpha})\mathrm{d}t \\ \mathbf{M}\boldsymbol{\psi} = p(t, \theta) \end{array}$$

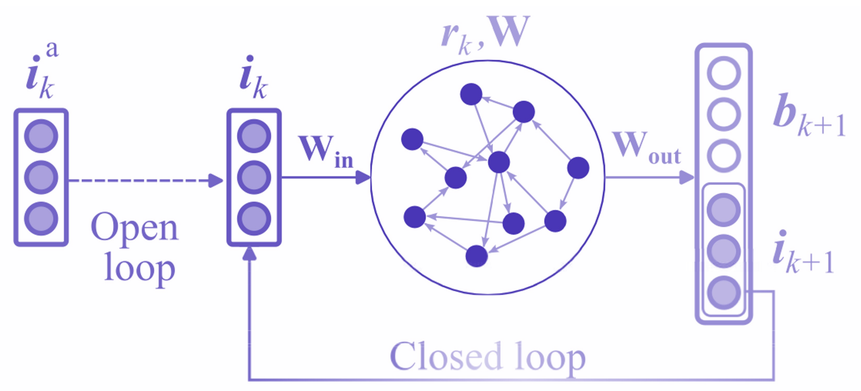
2. Raw acoustic pressure: data from four microphones, but sparse and noisy with a measurement shift.

$\boldsymbol{d} = \boldsymbol{p}(t, \boldsymbol{\theta}_{\mathrm{mic}} = \{0°, 60°, 120°, 240°\})$





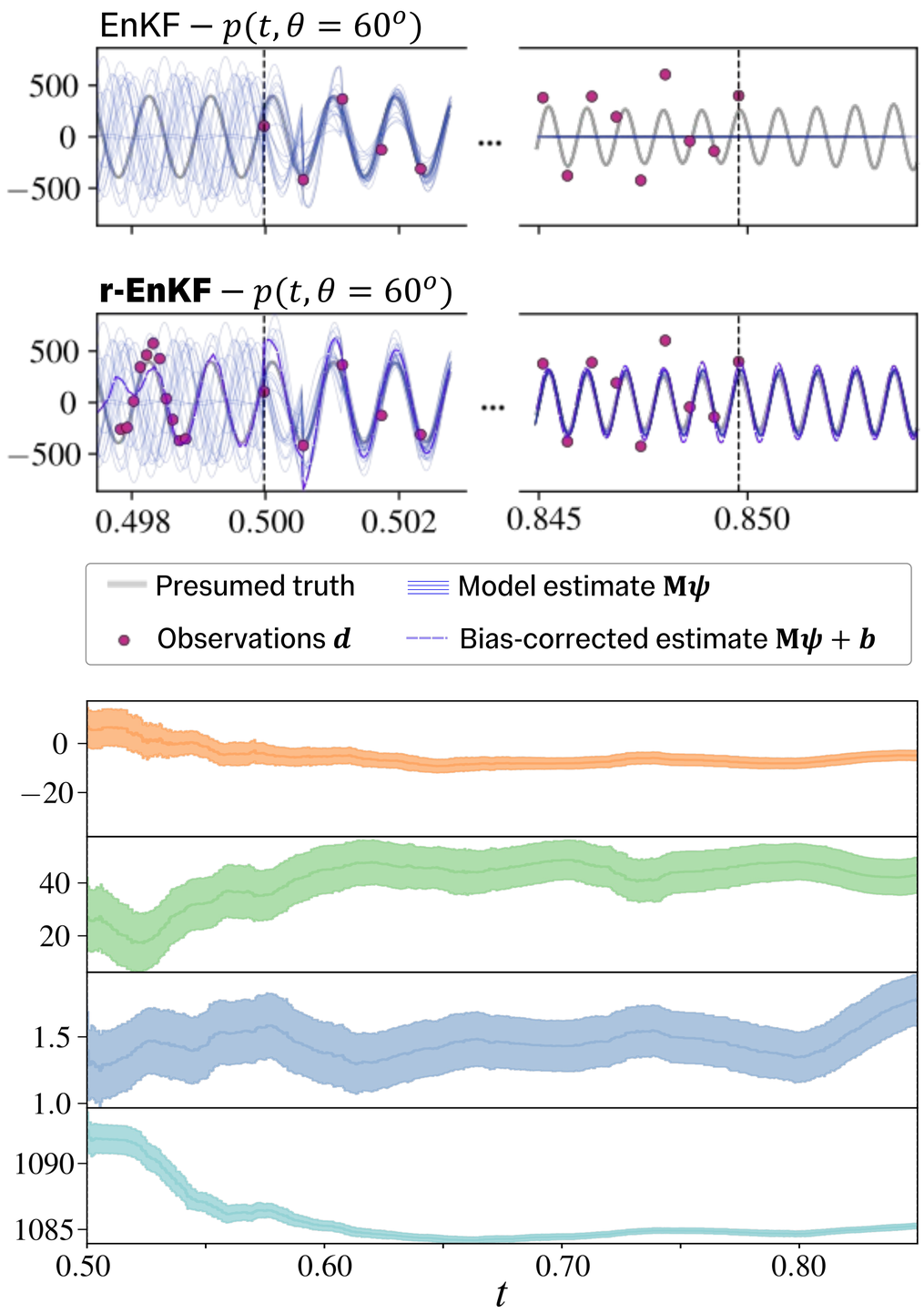
3. Echo state network (ESN): reservoir computer trained to estimate in real time the bias $\boldsymbol{b}$ and the measurement shift $\boldsymbol{b}_d$ from the innovations $\boldsymbol{i} = \boldsymbol{d} - \mathbf{M}\boldsymbol{\psi} \approx \boldsymbol{b} + \boldsymbol{b}_d$



4. The regularized bias-aware ensemble Kalman filter (r-EnKF [1,2]): a data assimilation tool to fuse models, data and uncertainties in real time.

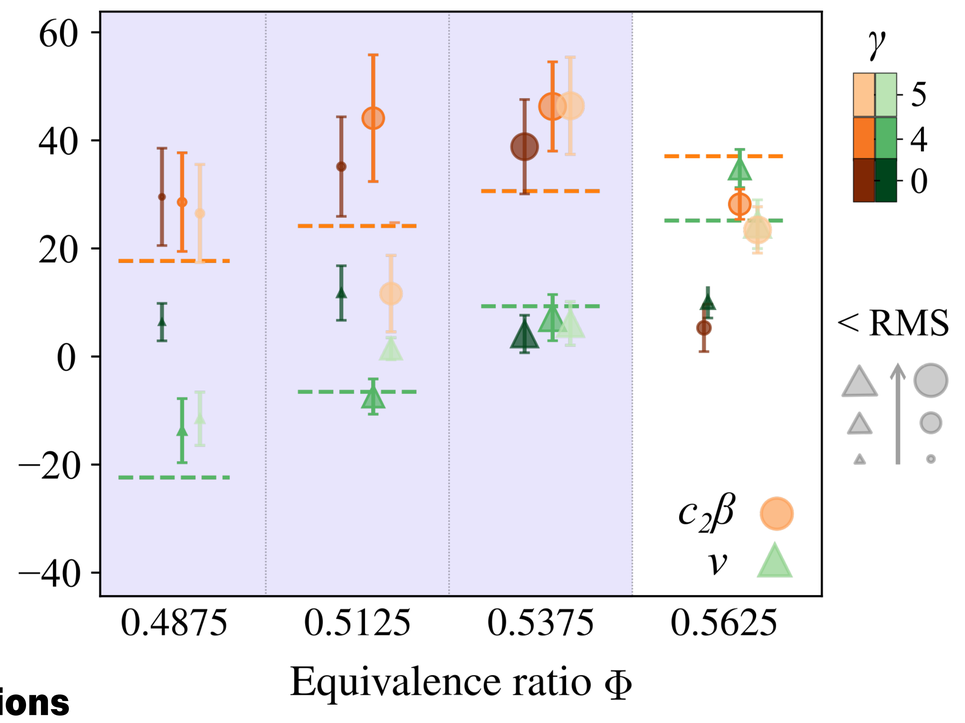
$$\mathcal{J}(\boldsymbol{\psi}_j) = \underbrace{||\boldsymbol{\psi}_j - \boldsymbol{\psi}_j^{\mathrm{f}}||^2_{\mathbf{C}^{-1}_{\psi\psi}}}_{\text{Prior}} + \underbrace{||\mathbf{M}\boldsymbol{\psi}_j + \boldsymbol{b} - \boldsymbol{d} - \boldsymbol{b}_d||^2_{\mathbf{C}^{-1}_{dd}}}_{\text{Bias-corrected likelihood}} + \underbrace{\gamma||\mathbf{b}||^2_{\mathbf{C}^{-1}_{bb}}}_{\text{Bias regularization}}$$

State, parameter and bias estimation



Generalizability study

We test the digital twin in unseeing dynamics: the ESN is trained only on data from the first three equivalence ratios Φ.



Conclusions

- We propose a digital twin framework to estimate in real time model parameters, states and modelling errors (i.e., biases).
- Key to the digital twin is the regularized bias-aware ensemble Kalman filter (r-EnKF).
- We create a digital twin of an annular combustor by combining pressure data, a physical low-order model, and an ESN to estimate the biases.
- The r-EnKF recovers physical solutions in real time in contrast to the classical bias-unaware EnKF.
- The digital twin generalizes to unseen operating conditions.
- The digital twin learns in real time more accurate models than the state-of-the-art, which are offline models.

[1]Nóvoa, Noiray, Dawson & Magri (2024). JFM, *To appear.*
[2]Nóvoa, Racca & Magri (2023). CMAME, *418, 116502.*

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