REMIND learning cost

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1 Naming

Concept	**Symbol**	**Name in fm_dataglob**
Investment costs (IC)	I	vm_costTeCapital
Initial IC	I_0	inco0
IC for doubled capacity	I_d	–
Floor cost (FC)	F	floorcost
Learnable investment costs	I'	incolearn
Cumulative capacity	C	vm_capCum
Initial cumulative capacity	C_0	ccap0
Observed learning rate	λ	learn
Learning rate with FC	λ'	–
Learning exponent	b	learnExp_woFC
Learning exponent with FC	b'	learnExp_wFC
Learning factor	a	learnMult_woFC
Learning factor with FC	a'	learnMult_wFC

2 Learning laws

In REMIND we use "one-factor learning curve" (or "experience curve"), see [2]. This widely-used formulation is derived from empirical observations across a variety of energy technologies that frequently indicate a log-linear relationship between the unit cost of the technology and its cumulative output (production) or installed capacity (see for example empirical paper [1]). With constants a and b representing the unit cost of the first unit and the rate of cost reduction, this model can be expressed in its simplest form as:

$$I = a \times C^b \tag{1}$$

Applying this law on the initial conditions, when investment costs are I_0 for a cumulative capacity C_0, we obtain:

$$a = \frac{I_0}{C_0^b} \tag{2}$$

Learning rate λ is defined as the fractional reduction in cost associated with a doubling of cumulative capacity. Let I_0 be the initial cost when cumulative capacity is C_0, and I_d be the cost when cumulative capacity is $C_d = 2 \times C_0$, then the learning rate is defined as:

$$\lambda = 1 - \frac{I_d}{I_0} \in [0, 1] \tag{3}$$

With $I_d = a \times (2C_0)^b = a \times C_0^b \times 2^b = I_0 \times 2^b$, we obtain:

$$\lambda = 1 - 2^b \qquad \text{or} \qquad b = \log_2(1 - \lambda) \tag{4}$$

Note that the rate of cost reduction b is not equal to learning rate λ.

Each time the capacity doubles, the investment costs are multiplied by $1-\lambda$. Hence **Wright's law** relating investment cost I and cumulative capacity C:

$$\frac{I}{I_0} = (1 - \lambda)^{\log_2\left(\frac{C}{C_0}\right)} = \left(\frac{C}{C_0}\right)^{\log_2(1-\lambda)} \tag{5}$$

3 Learning with floor cost

Now suppose there is a floor cost F such that $I \geq F \geq 0$, irrespective of the capacity. Then the learning only applies to learnable costs $I' = I - F$. The "observed" learning, that we could measure without knowing the floor cost, is now different from the mathematical learning parameters. Several assumptions are possible to match these two concepts.

3.1 Equal cost reduction when doubling capacity

Here we assume that the learning happens with a distinct learning rate λ', in order to achieve the same cost reduction for a doubling of capacity. Wright's law now reads:

$$\frac{I'}{I'_0} = \left(\frac{C}{C_0}\right)^{\log_2(1-\lambda')} \qquad \text{or} \qquad \frac{I - F}{I_0 - F} = \left(\frac{C}{C_0}\right)^{\log_2(1-\lambda')} \tag{6}$$

For a doubling of capacity, i.e. $C_d = 2C_0$, this law gives $\frac{I_d - F}{I_0 - F} = 1 - \lambda'$, while the observed learning rate still means $\lambda = 1 - \frac{I_d}{I_0}$. Those two equations combine into:

$$\lambda' = \frac{I_0}{I_0 - F}\lambda \tag{7}$$

The learning curve of Equation (1) with floor costs reads:

$$I' = a' \times C^{b'} \tag{8}$$

With

$$a' = \frac{I_0 - F}{C_0^{b'}} \tag{9}$$

$$b' = \log_2(1 - \lambda') = \log_2(1 - \frac{I_0}{I_0 - F}\lambda) \tag{10}$$

3.2 Equal initial slope of cost reduction

Another possibility is to ensure that, as the capacity increases, the investment cost decrease with the same slope as with the observed learning. In that case, we may want the slopes with or without floor cost to be the same at the beginning, when cumulative capacity is just above its initial value.

Using $\gamma = C/C_0$, we have the two equations

$$I = a \times C^b = I_0 \times \left(\frac{C}{C_0}\right)^b = I_0 \times \gamma^b \tag{11}$$

$$I' = a' \times C^{b'} = I'_0 \times \left(\frac{C}{C_0}\right)^{b'} = (I_0 - F) \times \gamma^{b'} \tag{12}$$

We are interested in the slope of these two functions, which are mathematically given by the derivative of I and I' with respect to γ:

$$\frac{dI}{d\gamma} = I_0 \times b \times \gamma^{b-1} \tag{13}$$

$$\frac{dI'}{d\gamma} = (I_0 - F) \times b' \times \gamma^{b'-1} \tag{14}$$

For the two curves to have the same slope at the beginning, when C is slightly higher than C_0, we want the two derivatives to be equal when $\gamma = 1$. This means $I_0 \times b = (I_0 - F) \times b'$, that we rewrite as:

$$b' = \frac{I_0}{I_0 - F} b \tag{15}$$

4 Remind equations

In Remind `fm_dataglob`, external data provides the observed learning rate `learn` (λ), the initial investment costs `inco0` (I_0), the learnable cost `incolearn` ($I'_0 = I_0 - F$) and the cumulative capacity in 2015 `ccap0` (C_0).

Let us list the equations as written in Remind, and their corresponding mathematical expression. The investment costs equation (after 2050 when the regional differentiation stops) is in `core/equations.gms`, while the learning factor equation and the learning exponent equation are in `core/datainput.gms`.

```
vm_costTeCapital =
    fm_dataglob("learnMult_wFC")
    * vm_capCum ** fm_dataglob("learnExp_wFC")
    + pm_data(regi,"floorcost")
```

$$I = a' \times C^{b'} + F \tag{16}$$

```
fm_dataglob("learnMult_wFC") =
    fm_dataglob("incolearn")
    / (fm_dataglob("ccap0") ** fm_dataglob("learnExp_wFC"))
```

$$a' = \frac{I'_0}{C_0^{b'}} \tag{17}$$

```
fm_dataglob("learnExp_wFC") =
    fm_dataglob("inco0") / fm_dataglob("incolearn")
    * log(1-fm_dataglob("learn"))/log(2)
```

$$b' = \frac{I_0}{I'_0} \times \log_2(1 - \lambda) \tag{18}$$

Equation (16) is equivalent to Equation (8), and Equation (17) is equivalent to Equation (9). As for Equation (18), it tells us which of the floor cost assumptions has been chosen in Remind: it is different from Equation (10), where the factor $\frac{I_0}{I_0 - F}$ is located within the logarithm, but it is equivalent to Equation (15). In conclusion, Remind code has chosen the assumption whereby the learning starts with the same slope regardless of the floor cost.

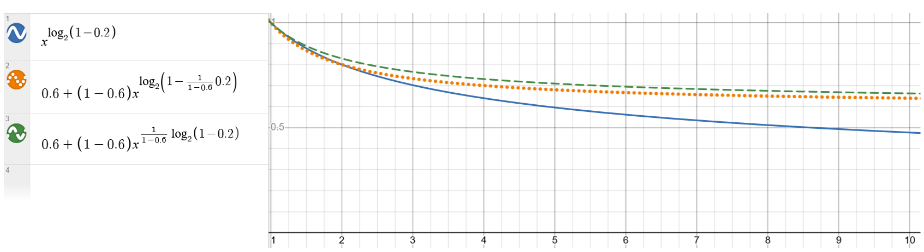

Figure 1: Comparison of three learning curves, assuming floor cost $F = 0.6I_0$ and $\lambda = 0.2$. Horizontal axis: relative cumulative capacity C/C_0. Vertical axis: relative investment cost reduction I/I_0. Solid blue: learning curve without floor cost. Dotted orange: learning curve with floor cost assuming same cost reduction for capacity doubling (see intersection of blue and orange for $x = 2$). Dashed green: learning curve with floor cost assuming same initial slope (in $x = 1$, blue and green have the same slope).

References

[1] Alan McDonald and Leo Schrattenholzer. Learning rates for energy technologies. *Energy Policy*, 29(4):255–261, 2001.

[2] Edward S. Rubin, Inês M.L. Azevedo, Paulina Jaramillo, and Sonia Yeh. A review of learning rates for electricity supply technologies. *Energy Policy*, 86:198–218, 2015.