CPSC 340: Machine Learning and Data Mining

Deep Learning

Spring 2022 (2021W2)

Admin

- Course surveys
  - Please fill them out
  - We care deeply about your education, so we take them very seriously
  - You will be able to evaluate the class overall, and then Mijung and I separately
  - Please use the text boxes to also let us know about the "lecture specialization experiment" [where we each specialized in half the lectures]
  - As always, please remember we're real people, so both praise and critical feedback are great. Please avoid personal, hurtful, or unconstructive negative comments.

- A5 deadline tonight
- A6 out soon: by Monday at the latest, due April 8 (our last class)

End of Part 4: Key Concepts

- We discussed linear latent-factor models:

$$f(W,Z) = \sum_{i=1}^{n} \sum_{j=1}^{d} (\langle w^j, z_i \rangle - x_{ij})^2$$

$$= \sum_{i=1}^{n} \|W^T z_i - x_i\|^2$$

$$= \|ZW - X\|_F^2$$

- Represent 'X' as linear combination of latent factors 'w_c'.
  - Latent features 'z_i' give a lower-dimensional version of each 'x_i'.
  - When k=1, finds direction that minimizes squared orthogonal distance.
- Applications:
  - Outlier detection, dimensionality reduction, data compression, features for linear models, visualization, factor discovery, filling in missing entries.

End of Part 4: Key Concepts

- We discussed linear latent-factor models:

$$f(W,Z) = \sum_{i=1}^{n} \sum_{j=1}^{d} (\langle w^j, z_i \rangle - x_{ij})^2$$

- Principal component analysis (PCA):
  - Often uses orthogonal factors and fits them sequentially (via SVD).
- Non-negative matrix factorization:
  - Uses non-negative factors giving sparsity.
  - Can be minimized with projected gradient.
- Many variations are possible:
  - Different regularizers (sparse coding) or loss functions (robust/binary PCA).
  - Missing values (recommender systems) or change of basis (kernel PCA).

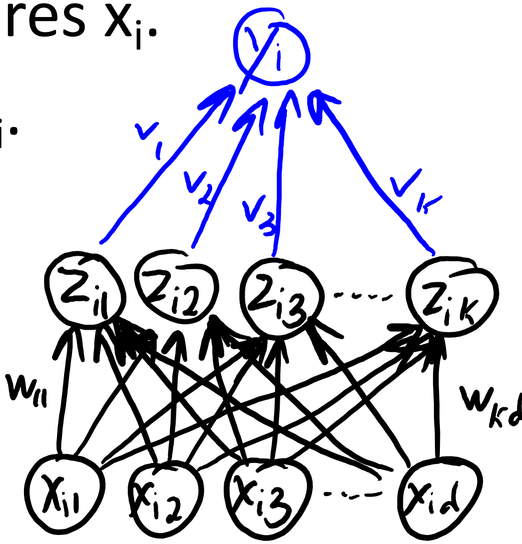
bonus!

End of Part 4: Key Concepts

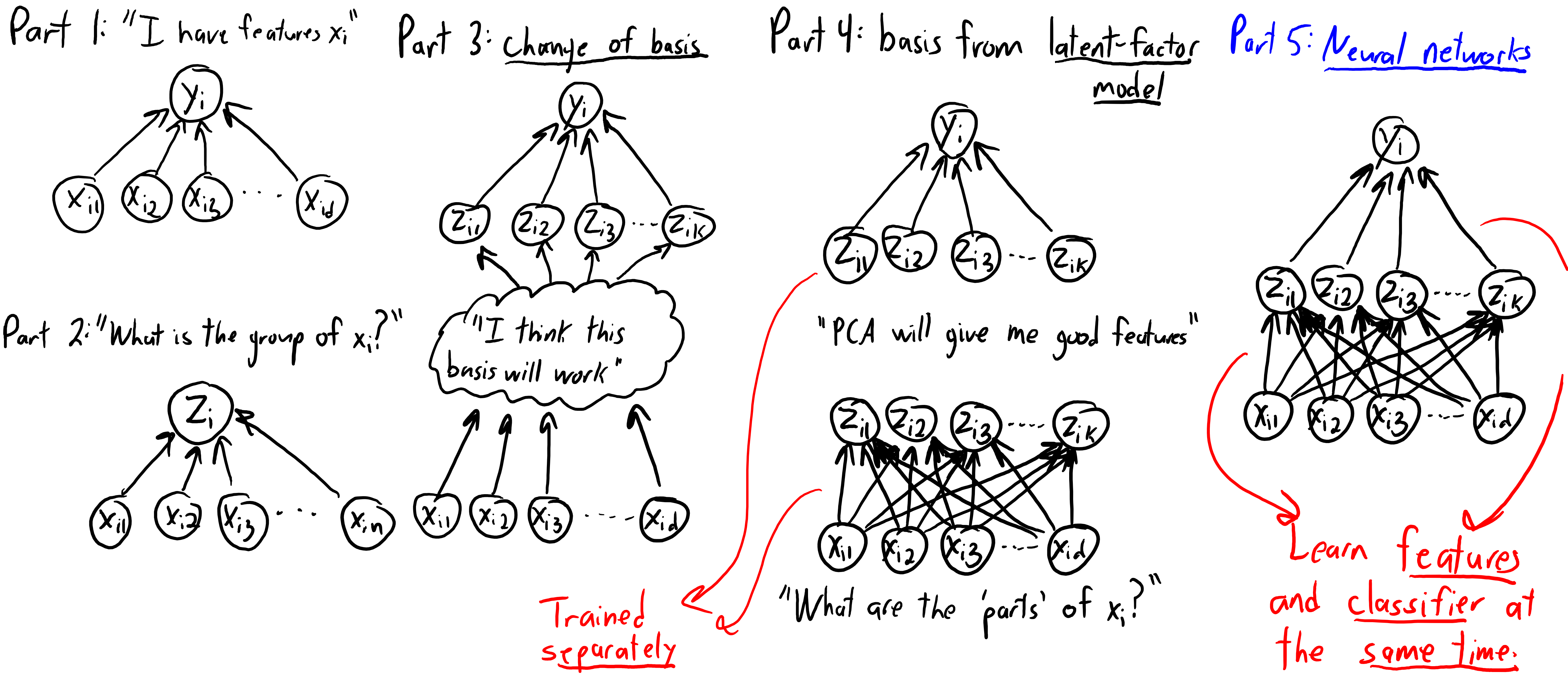
- We didn't really discuss multi-dimensional scaling (MDS):
  - Non-parametric method for high-dimensional data visualization.
  - Tries to match distance/similarity in high-/low-dimensions.
    - "Gradient descent on scatterplot points".
- Main challenge in MDS methods is "crowding" effect:
  - Methods focus on large distances and lose local structure.
- Common solutions:
  - Sammon mapping: use weighted cost function.
  - ISOMAP: approximate geodesic distance using via shortest paths in graph.
  - T-SNE: give up on large distances and focus on neighbour distances.
- Word2vec is a recent MDS method giving better "word features".

Supervised Learning Roadmap

- Part 1: "Direct" Supervised Learning.
  - We learned parameters 'w' based on the original features x_i and target y_i.
- Part 3: Change of Basis.
  - We learned parameters 'v' based on a change of basis z_i and target y_i.
- Part 4: Latent-Factor Models.
  - We learned parameters 'W' for basis z_i based on only on features x_i.
  - You can then learn 'v' based on change of basis z_i and target y_i.
- Part 5: Neural Networks.
  - Jointly learn 'W' and 'v' based on x_i and y_i.
  - **Learn basis z_i that is good for supervised learning.**



A Graphical Summary of CPSC 340 Parts 1-5



Notation for Neural Networks (MEMORIZE)

We have our usual supervised learning notation:

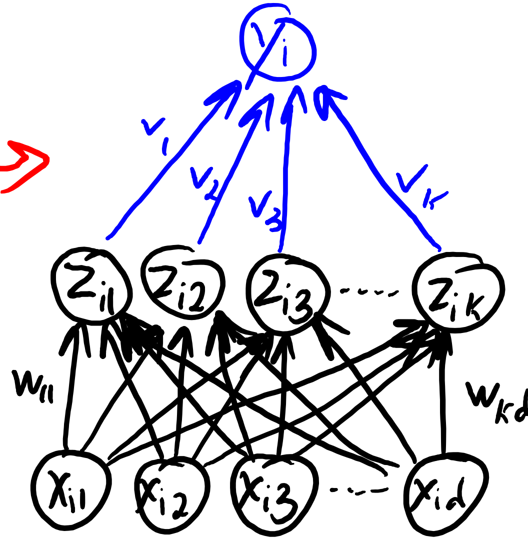
$$X = \underbrace{\begin{bmatrix} — & x_1^T & — \\ — & x_2^T & — \\ & \vdots & \\ — & x_n^T & — \end{bmatrix}}_{n \times d} \qquad y = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{n \times 1}$$

We have our latent features:

$$Z = \underbrace{\begin{bmatrix} — & z_1^T & — \\ — & z_2^T & — \\ & \vdots & \\ — & z_n^T & — \end{bmatrix}}_{n \times k}$$

We have two sets of parameters:

$$v = \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix}}_{k \times 1} \qquad W = \underbrace{\begin{bmatrix} — & w_1 & — \\ — & w_2 & — \\ & \vdots & \\ — & w_k & — \end{bmatrix}}_{k \times d}$$



Linear-Linear Model

- Natural choice: linear latent-factor model with linear regression.

Use features from latent-factor model: $z_i = Wx_i$

Make predictions using a linear model: $y_i = v^T z_i$

- We want to train 'W' and 'v' jointly, so we could minimize:

$$f(W,v) = \frac{1}{2}\sum_{i=1}^{n}\underbrace{(v^T z_i - y_i)^2}_{\substack{\text{linear regression} \\ \text{with } z_i \text{ as features}}} = \frac{1}{2}\sum_{i=1}^{n}(v^T \underbrace{(Wx_i)}_{\substack{z_i \text{ come from} \\ \text{latent-factor model}}} - y_i)^2$$

- But this is just a linear model: $\hat{y}_i = v^T z_i = v^T(Wx_i) = \underbrace{(\overbrace{v^T W}^{1 \times d})}_{\text{some vector 'w'}} x_i = w^T x_i$

Introducing Non-Linearity

- To increase flexibility, something needs to be non-linear.
- Typical choice: transform z_i by non-linear function 'h'.

$$z_i = Wx_i \qquad \hat{y}_i = v^T h(z_i)$$

  – Here the function 'h' transforms 'k' inputs to 'k' outputs.
- Common choice for 'h': applying sigmoid function element-wise:

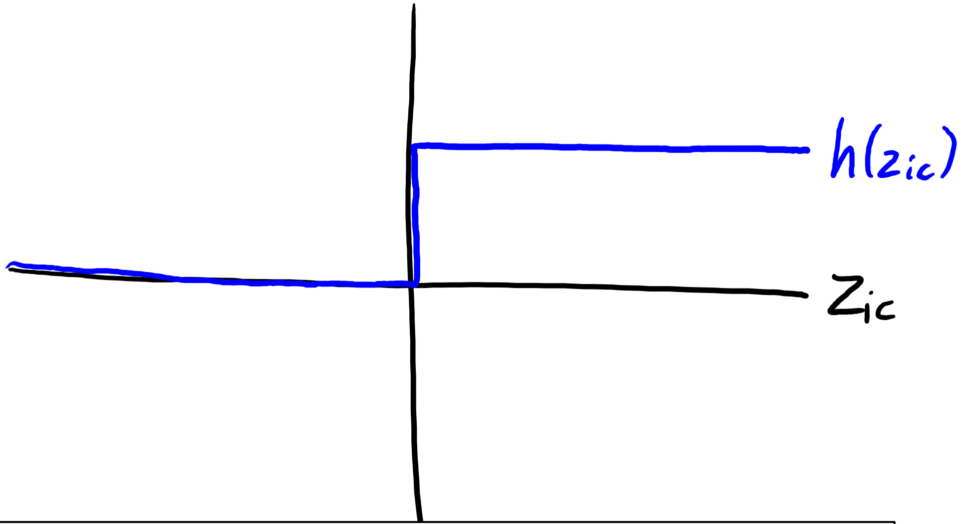
$$h(z_{ic}) = \frac{1}{1+\exp(-z_{ic})}$$

- So this takes the z_{ic} in $(-\infty,\infty)$ and maps it to $(0,1)$.
- This is called a "multi-layer perceptron" or a "neural network".

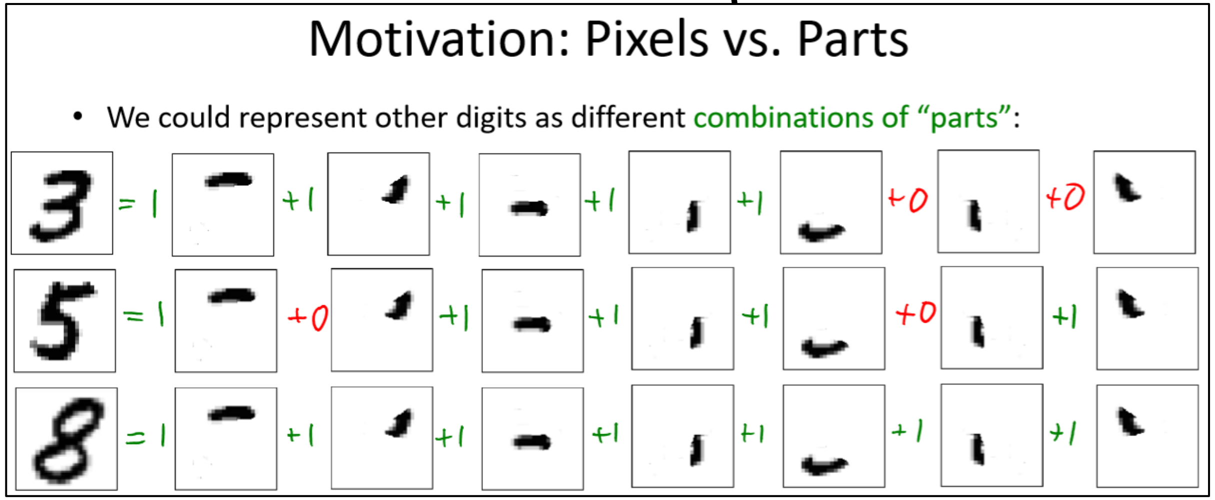
Why Sigmoid?

- Consider setting ‘h’ to define binary features z_i using:

$$h(z_{ic}) = \begin{cases} 1 & \text{if } z_{ic} \geq 0 \\ 0 & \text{if } z_{ic} < 0 \end{cases}$$



  – Each $h(z_i)$ can be viewed as binary feature.
    - “You either have this ‘part’ or you don’t have it.”
  – We can make 2^k objects by all the possible “part combinations”.



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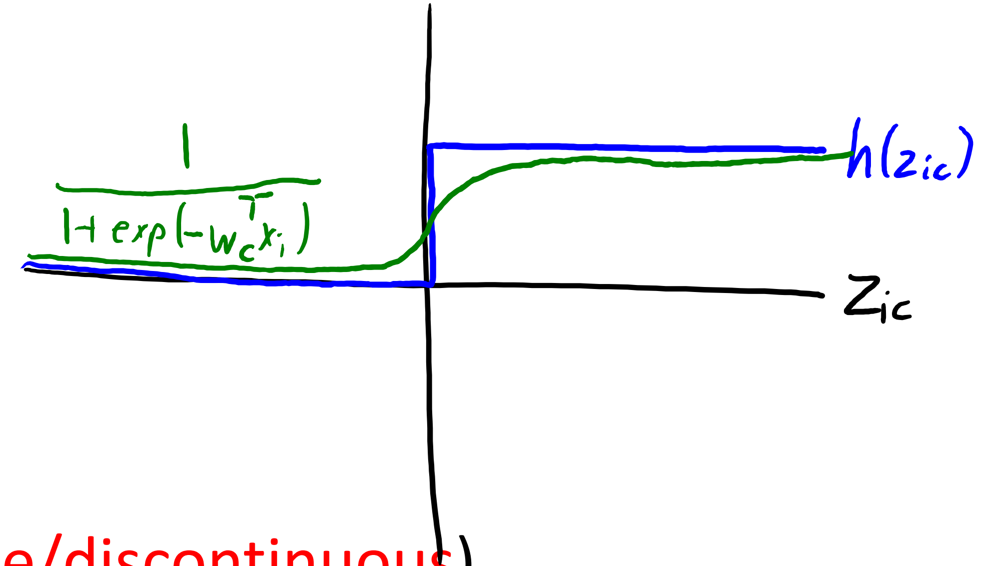
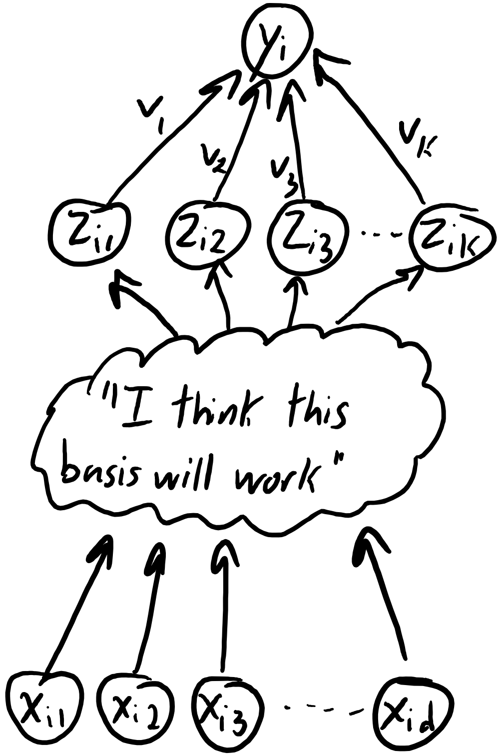


  – Each h(zi) can be viewed as binary feature.
    - “You either have this ‘part’ or you don’t have it.”
  – But this is hard to optimize (non-differentiable/discontinuous).
- Sigmoid is a smooth approximation to these binary features.
  – Non-parametric version is a universal approximator:
    - If ‘k’ grows appropriately with ‘n’, can model any continuous function.

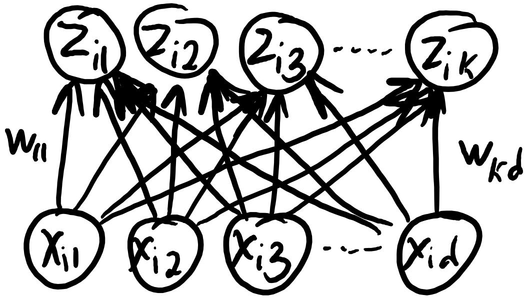
Supervised Learning Roadmap

Hand-engineered features:

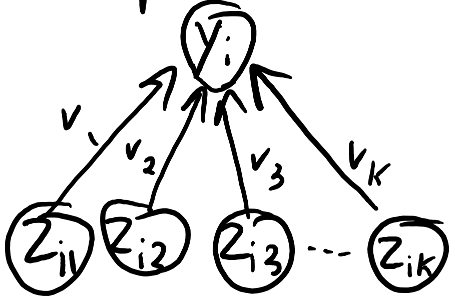


Requires domain knowledge and can be time-consuming

Learn a latent-factor model:

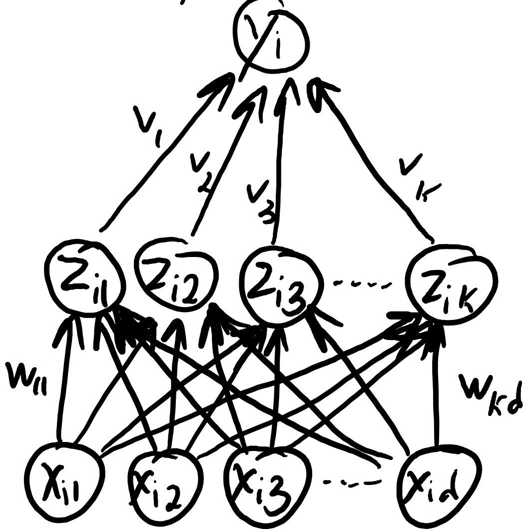


Use latent features in supervized model:



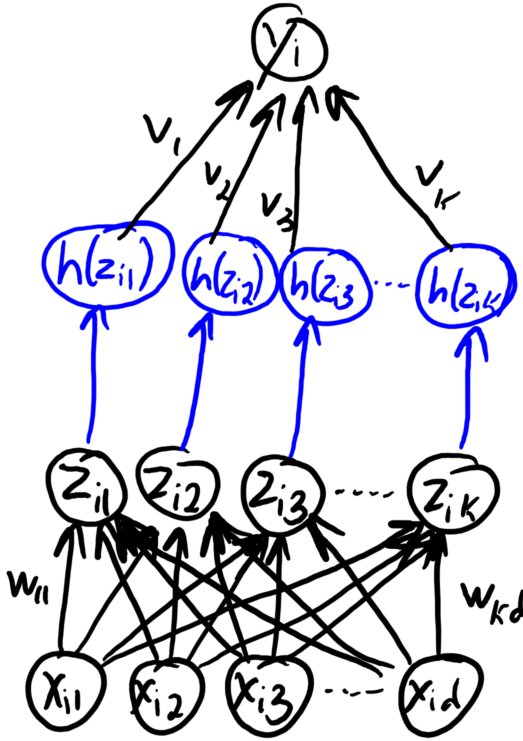
Good representation of x_i might be bad for predicting y_i

Learn 'w' and 'W' together:



But still gives a linear model.

Neural network:

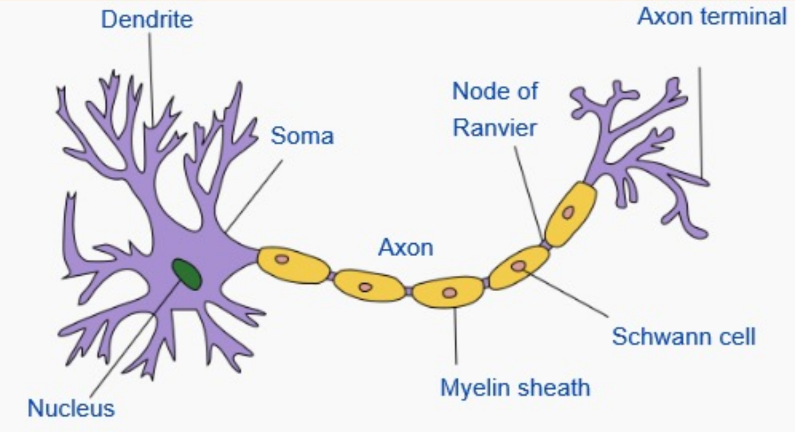


Extra non-linear transformation 'h'.

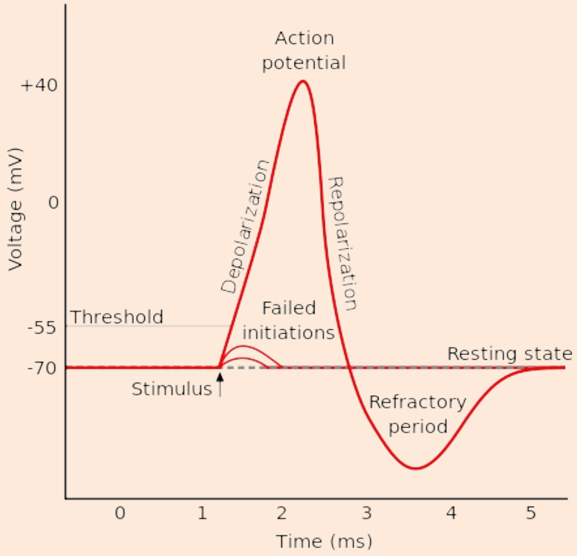
bonus!

Why “Neural Network”?

- Cartoon of “typical” neuron:



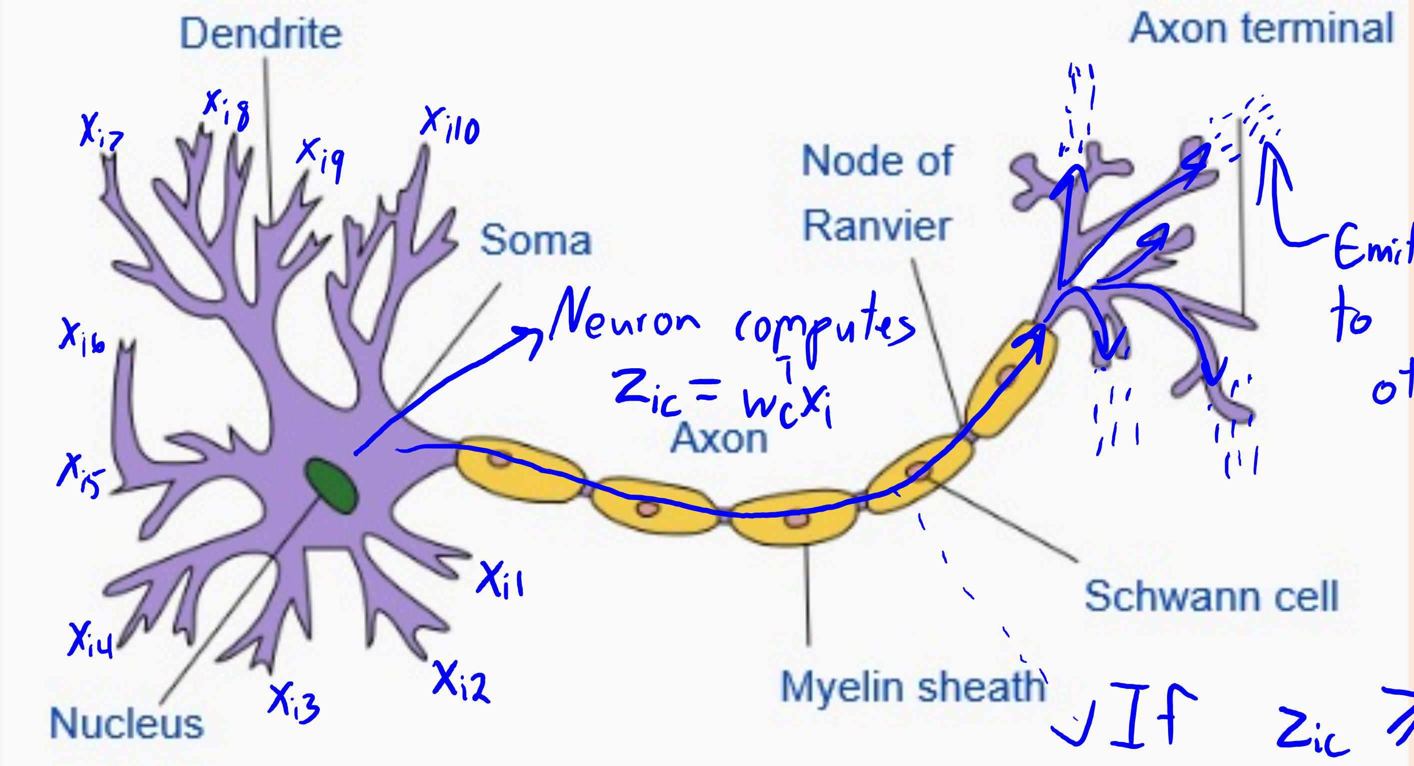
- Neuron has many “dendrites”, which take an input signal.
- Neuron has a single “axon”, which sends an output signal.
- With the right input to dendrites:
  - “Action potential” along axon (like a binary signal):



https://en.wikipedia.org/wiki/Neuron
https://en.wikipedia.org/wiki/Action_potential

bonus!

Why “Neural Network”?



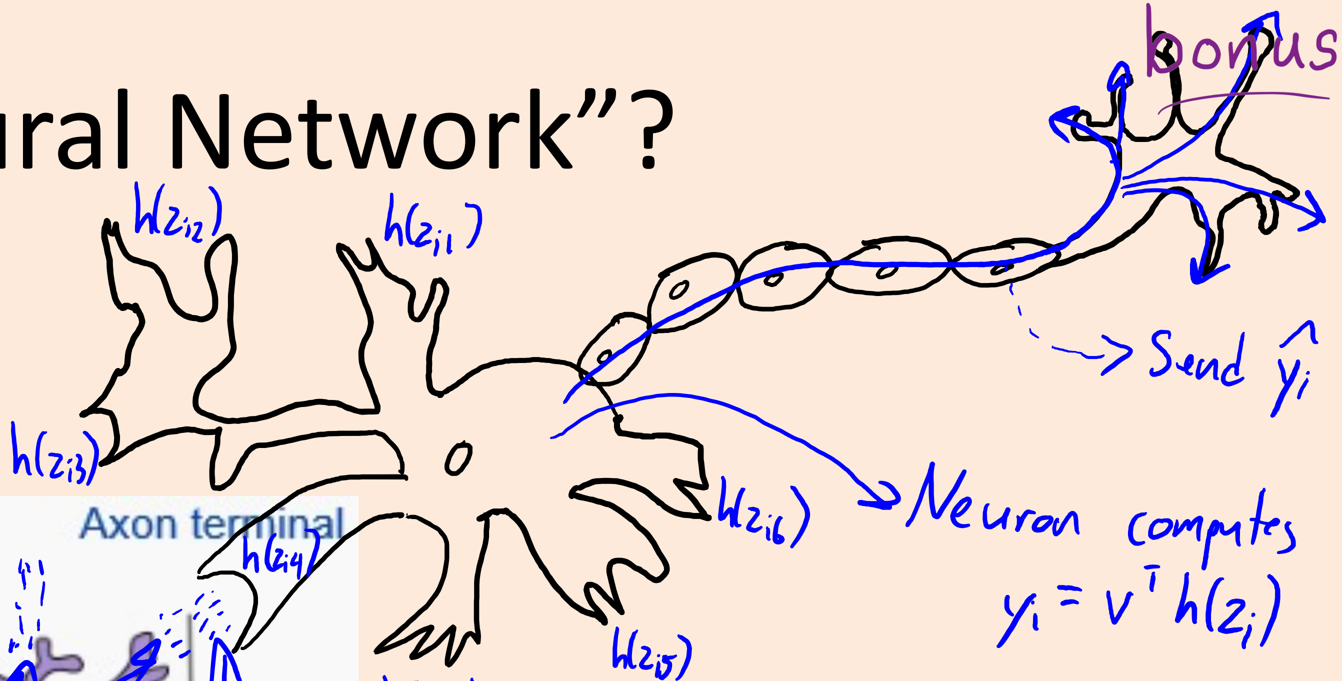
Emits "neurotransmitter" to send signal to other neurons.

If $z_{ic} \geq 0$ neuron sends signal along axon.

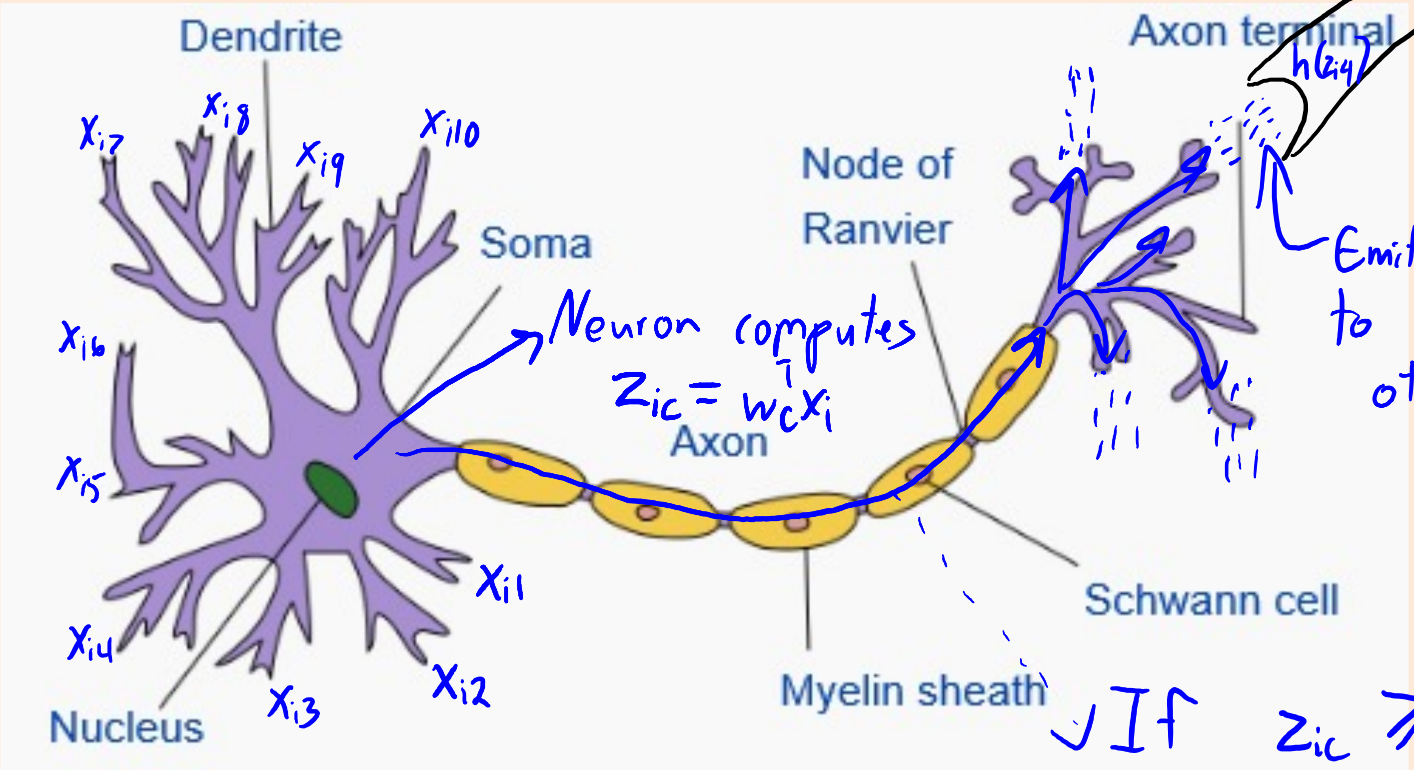
We approximate binary signal with $\frac{1}{1+\exp(-z_{ic})}$

https://en.wikipedia.org/wiki/Neuron

Why "Neural Network"?



Neuron computes $y_i = v^T h(z_i)$



Neuron computes $z_{ic} = w_c^T x_i$

Emits "neurotransmitter" to send signal to other neurons.

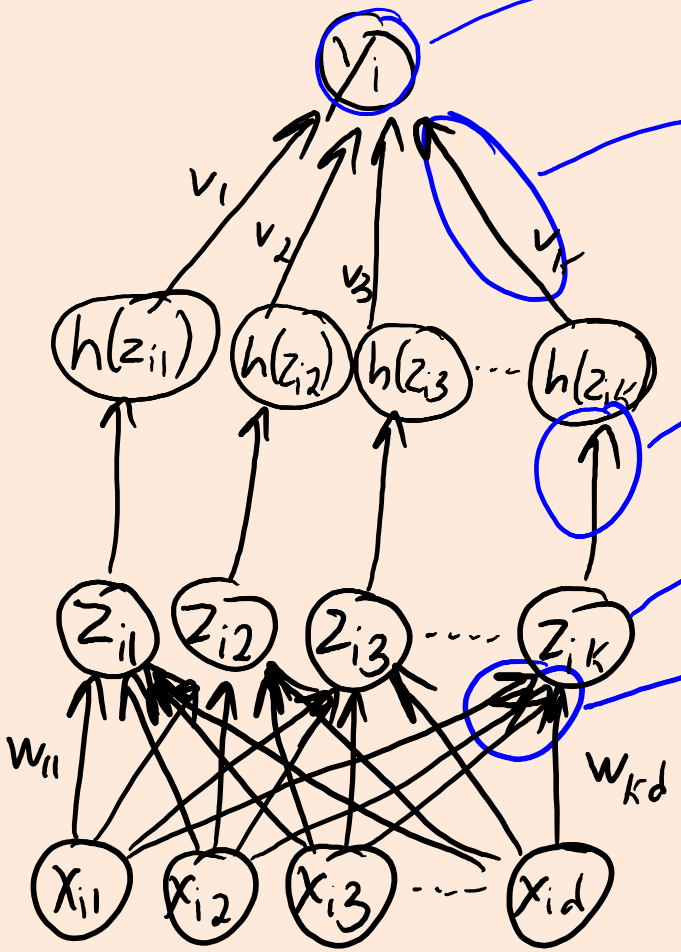
If $z_{ic} \geq 0$ neuron sends signal along axon.

We approximate binary signal with $\frac{1}{1+\exp(-z_{ic})}$

https://en.wikipedia.org/wiki/Neuron

bonus!

Why "Neural Network"?



Predictions based on aggregation $v^T h(Wx_i)$ at y_i "neuron"

Synapse between z_{ik} and y_i "neuron"

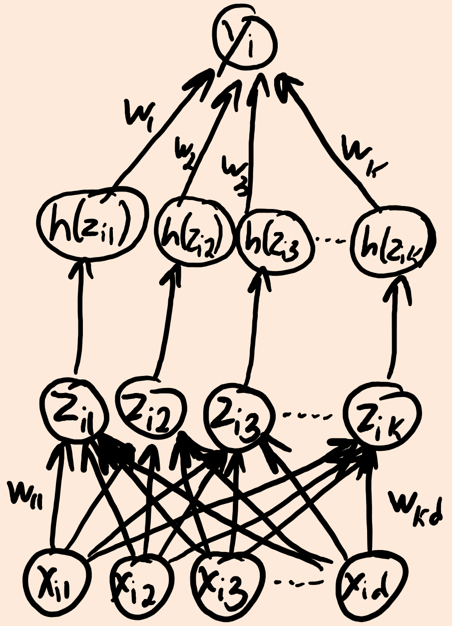
binary signal $h(w_c^T x_i)$ sent along "axon"

Neuron aggregates signals: $w_c^T x_i$

"dendrites" for z_{ik} "neuron" are reciving x_{ij} values

bonus!

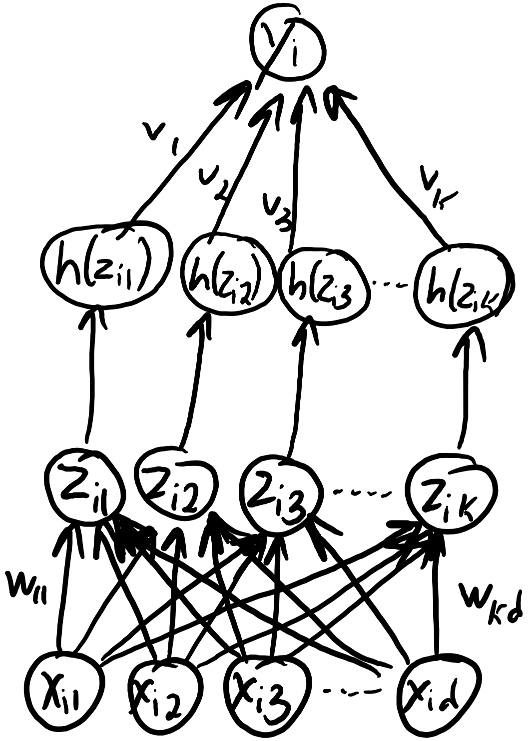
"Artificial" Neural Nets vs. "Real" Networks Nets



- Artificial neural network:
  - x_i is measurement of the world.
  - z_i is internal representation of world.
  - y_i is output of neuron for classification/regression.
- Real neural networks are more complicated:
  - Timing of action potentials seems to be important.
    - "Rate coding": frequency of action potentials simulates continuous output.
  - Neural networks don't reflect sparsity of action potentials.
  - How much computation is done inside neuron?
  - Brain is highly organized (e.g., substructures and cortical columns).
  - Connection structure changes.
  - Different types of neurotransmitters.

Deep Learning

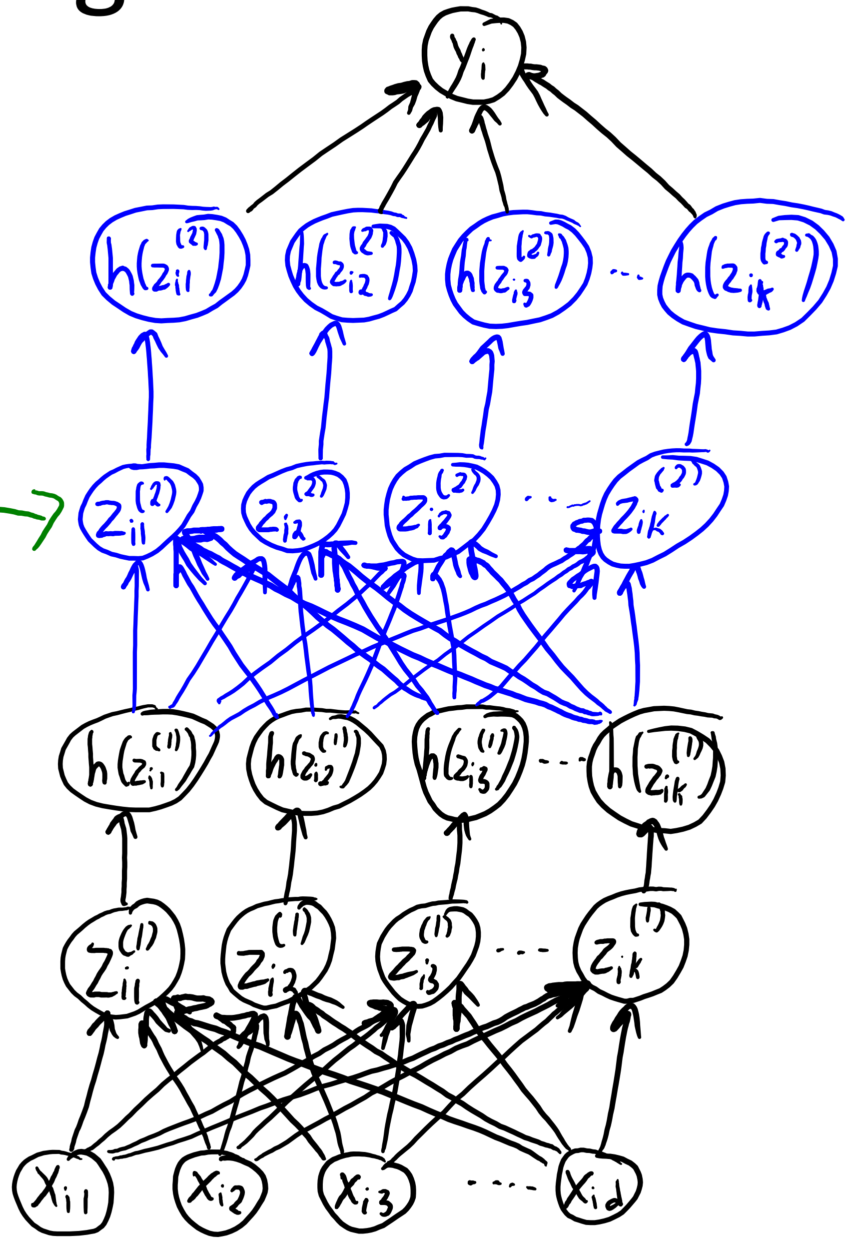
Neural network:



Deep learning:

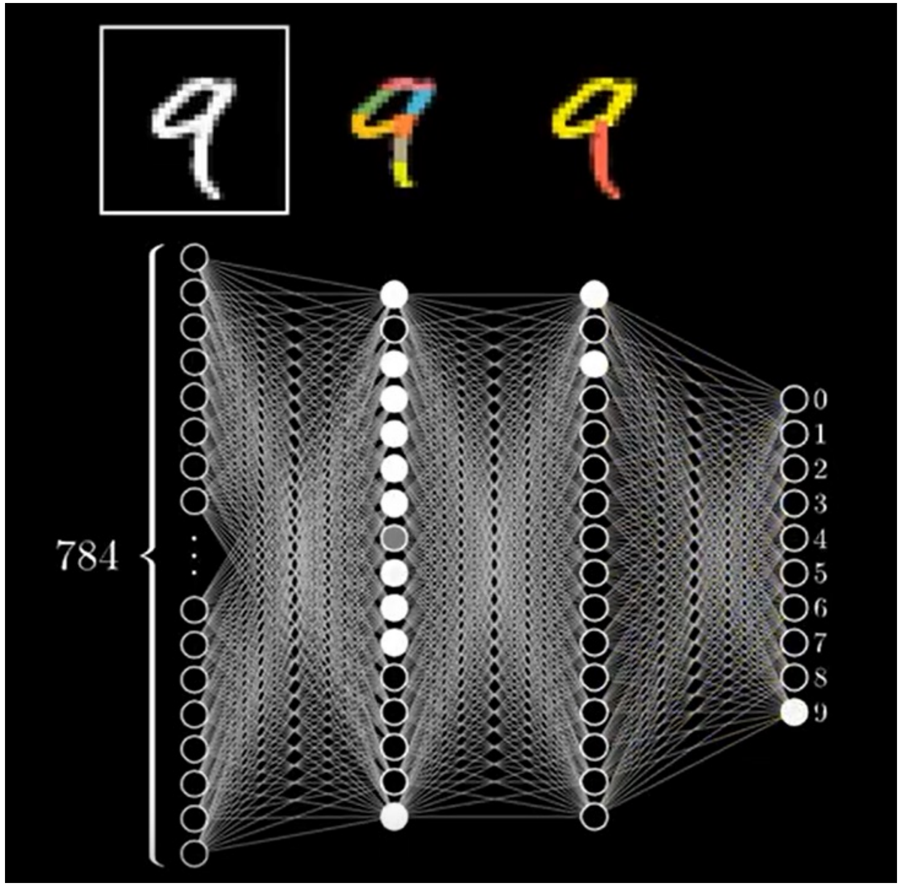
Second "layer" of latent features

You can add more "layers" to go "deeper"



"Hierarchies of Parts" Motivation for Deep Learning

- Each "neuron" might recognize a "part" of a digit.
  - "Deeper" neurons might recognize combinations of parts.
  - Represent complex objects as hierarchical combinations of re-useable parts (a simple "grammar").



- Watch the full video here:
  - https://www.youtube.com/watch?v=aircAruvnKk

- Theory:
  - 1 big-enough hidden layer already gives universal approximation.
  - But some functions require exponentially-fewer parameters to approximate with more layers (can fight curse of dimensionality).

Deep Learning

Linear model:

$$\hat{y}_i = w^T x_i$$

Neural network with 1 hidden layer:

$$\hat{y}_i = v^T h(\underbrace{W x_i}_{z_i})$$

Neural network with 2 hidden layers:

$$\hat{y}_i = v^T h(\underbrace{W^{(2)} h(\underbrace{W^{(1)} x_i}_{z_i^{(1)}})}_{z_i^{(2)}})$$

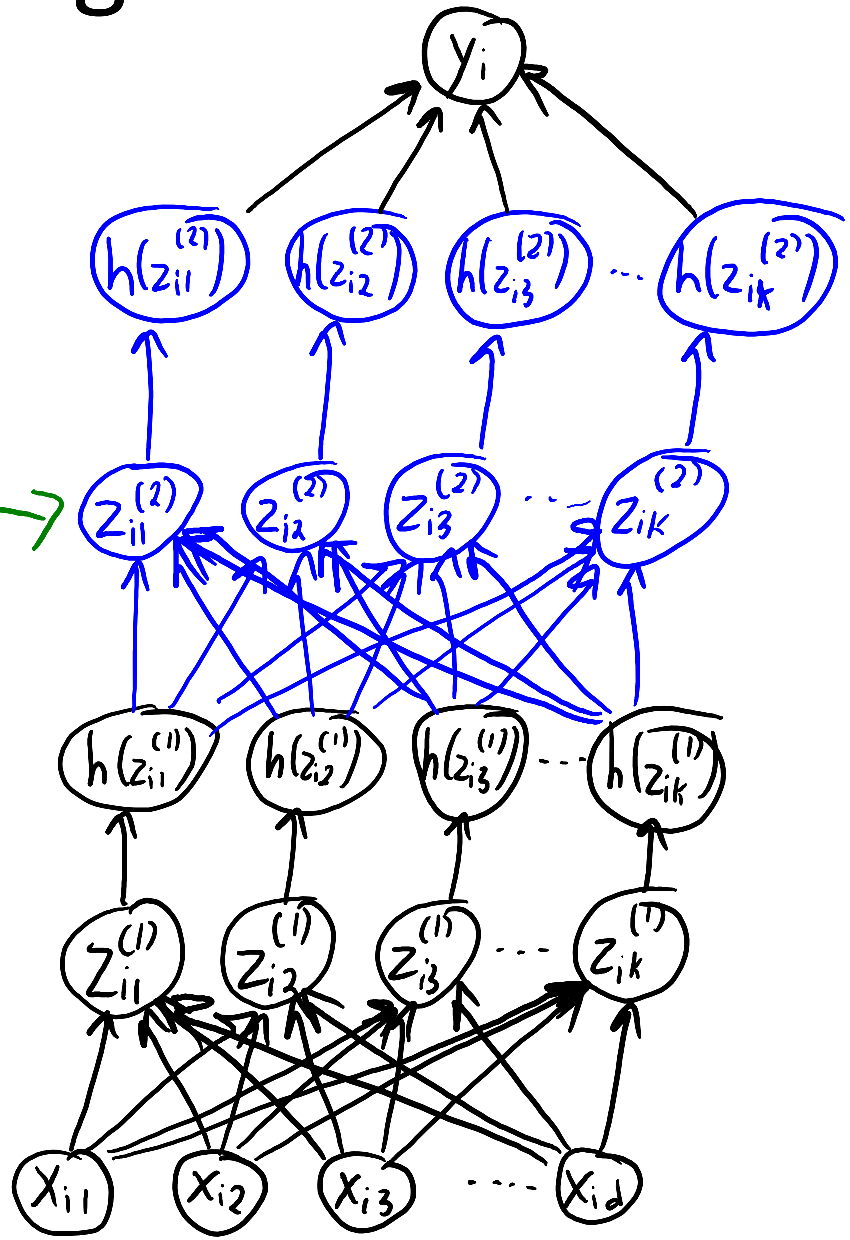
Neural network with 3 hidden layers

$$\hat{y}_i = v^T h(\underbrace{W^{(3)} h(\underbrace{W^{(2)} h(\underbrace{W^{(1)} x_i}_{z_i^{(1)}})}_{z_i^{(2)}})}_{z_i^{(3)}})$$

Deep learning:

Second "layer" of latent features

You can add more "layers" to go "deeper"



Deep Learning

- For 4 layers, we could write the prediction as:

$$\hat{y}_i = v^T h(W^{(4)} h(W^{(3)} h(W^{(2)} h(W^{(1)} x_i))))$$

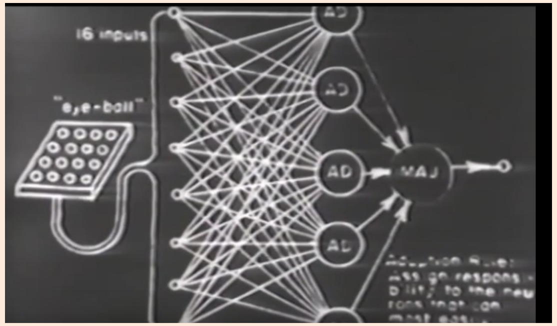
- For 'm' layers, we usually just say:

$$\hat{y}_i = v^T h(W^{(m)} h(\cdots h(W^{(1)} x_i)))$$

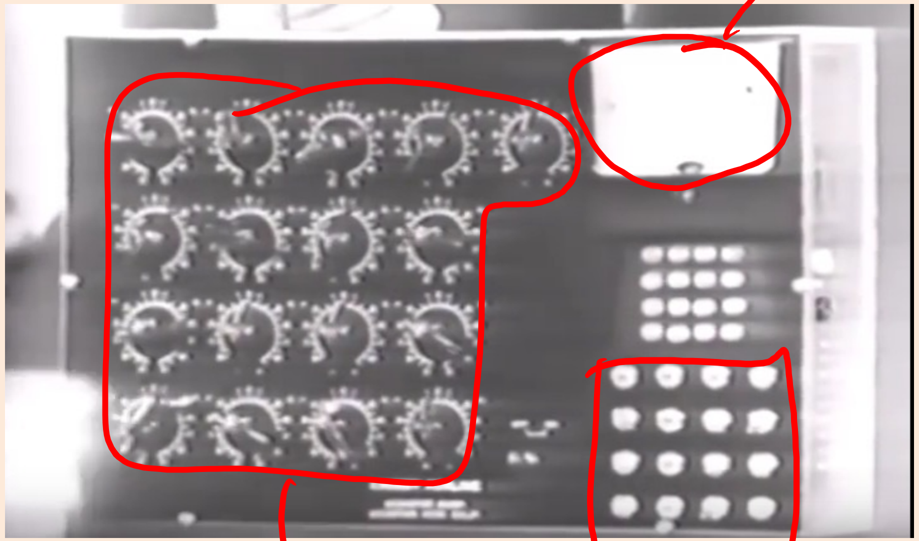
bonus!

ML and Deep Learning History

- 1950 and 1960s: Initial excitement.
  - Perceptron: linear classifier and stochastic gradient (roughly).
  - "the embryo of an electronic computer that [the Navy] expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence." New York Times (1958).
    - https://www.youtube.com/watch?v=IEFRtz68m-8
  - Object recognition assigned to students as a summer project



$w^T x_i$



w

x_i

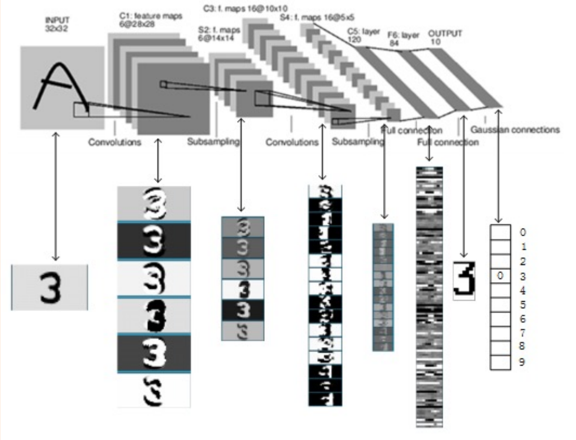
- Then drop in popularity:
  - Quickly realized limitations of linear models.

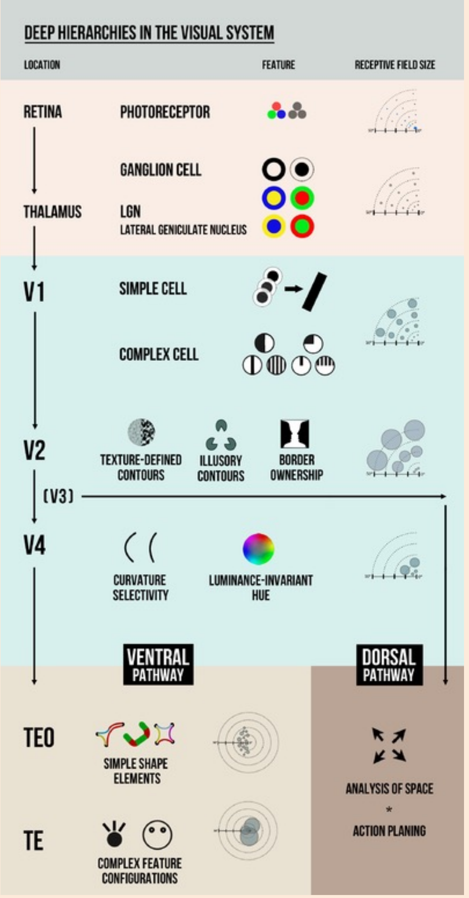
https://mitpress.mit.edu/books/perceptrons/

bonus!

ML and Deep Learning History

- 1970 and 1980s: Connectionism (brain-inspired ML)
  - Want "connected networks of simple units".
    - Use parallel computation and distributed representations.
  - Adding hidden layers z_i increases expressive power.
    - With 1 layer and enough sigmoid units, a universal approximator.
  - Success in optical character recognition.





https://en.wikibooks.org/wiki/Sensory_Systems/Visual_Signal_Processing
http://www.datarobot.com/blog/a-primer-on-deep-learning/
http://blog.csdn.net/strint/article/details/44163869

bonus!

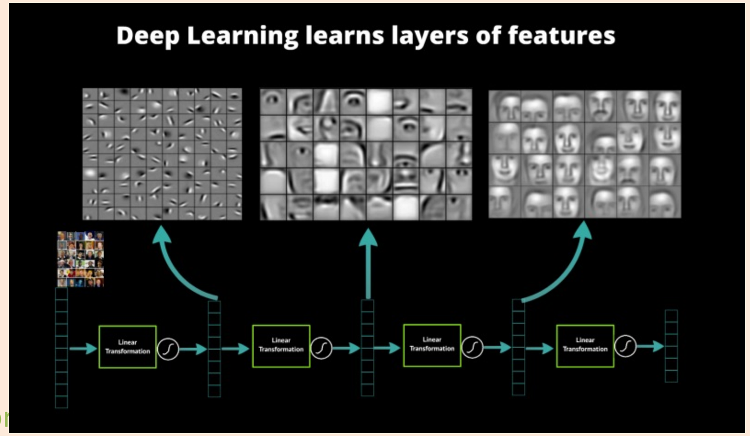
ML and Deep Learning History

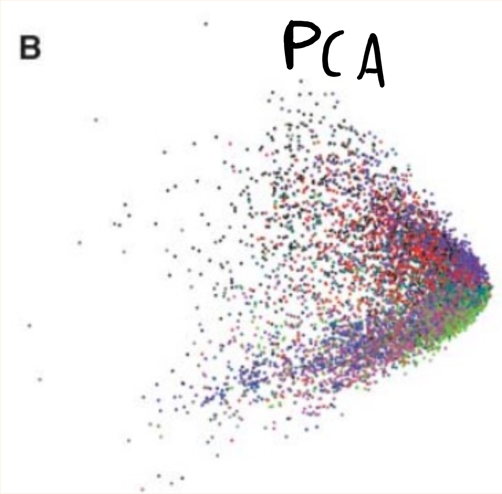
- 1990s and early-2000s: drop in popularity.
  - It proved really difficult to get multi-layer models working robustly.
  - We obtained similar performance with simpler models:
    - Rise in popularity of logistic regression and SVMs with regularization and kernels.
  - Lots of internet successes (spam filtering, web search, recommendation).
  - ML moved closer to other fields like numerical optimization and statistics.

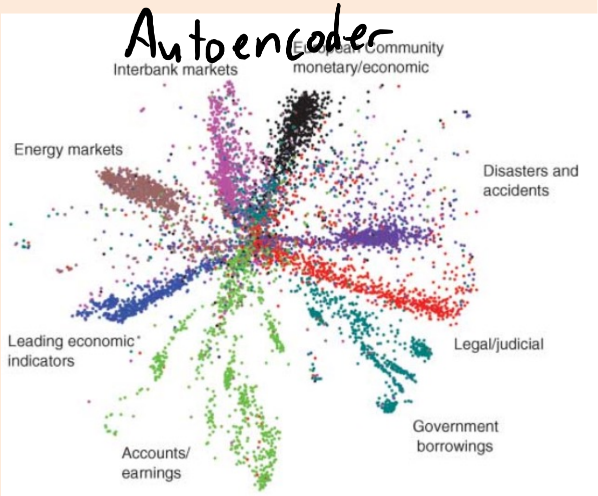
bonus!

ML and Deep Learning History

- Late 2000s: push to revive connectionism as "deep learning".
  - Canadian Institute For Advanced Research (CIFAR) NCAP program:
    - "Neural Computation and Adaptive Perception".
    - Led by Geoff Hinton, Yann LeCun, and Yoshua Bengio
    - Unsupervised successes: "deep belief networks" and "autoencoders".
    - Could be used to initialize deep neural networks.
    - https://www.youtube.com/watch?v=KuPai0ogiHk





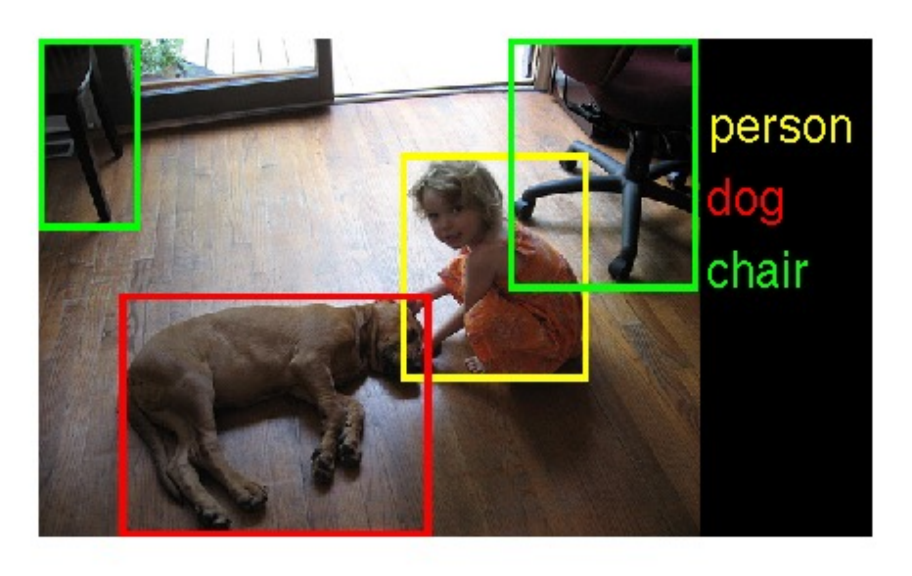


https://www.cs.tor

bonus!

2010s: DEEP LEARNING!!!

- Bigger datasets, bigger models, parallel computing (GPUs/clusters).
  - And some tweaks to the models from the 1980s.
- Huge improvements in automatic speech recognition (2009).
  - All phones now have deep learning.
- Huge improvements in computer vision (2012).
  - Changed computer vision field almost instantly.
  - This is now finding its way into products.



http://www.image-net.org/challenges/LSVRC/2014/

bonus!

2010s: DEEP LEARNING!!!

- Media hype:
  - "How many computers to identify a cat? 16,000"
    New York Times (2012).
  - "Why Facebook is teaching its machines to think like humans"
    Wired (2013).
  - "What is 'deep learning' and why should businesses care?"
    Forbes (2013).
  - "Computer eyesight gets a lot more accurate"
    New York Times (2014).

- 2015: huge improvement in language understanding.

Summary

- Neural networks learn features z_i for supervised learning.
- Sigmoid function avoids degeneracy by introducing non-linearity.
  - Universal approximator with large-enough 'k'.
- Biological motivation for (deep) neural networks.
- Deep learning considers neural networks with many hidden layers.
  - Can more-efficiently represent some functions.
- Unprecedented performance on difficult pattern recognition tasks.

- Next time:
  - Training deep networks.

bonus!

Multiple Word Prototypes

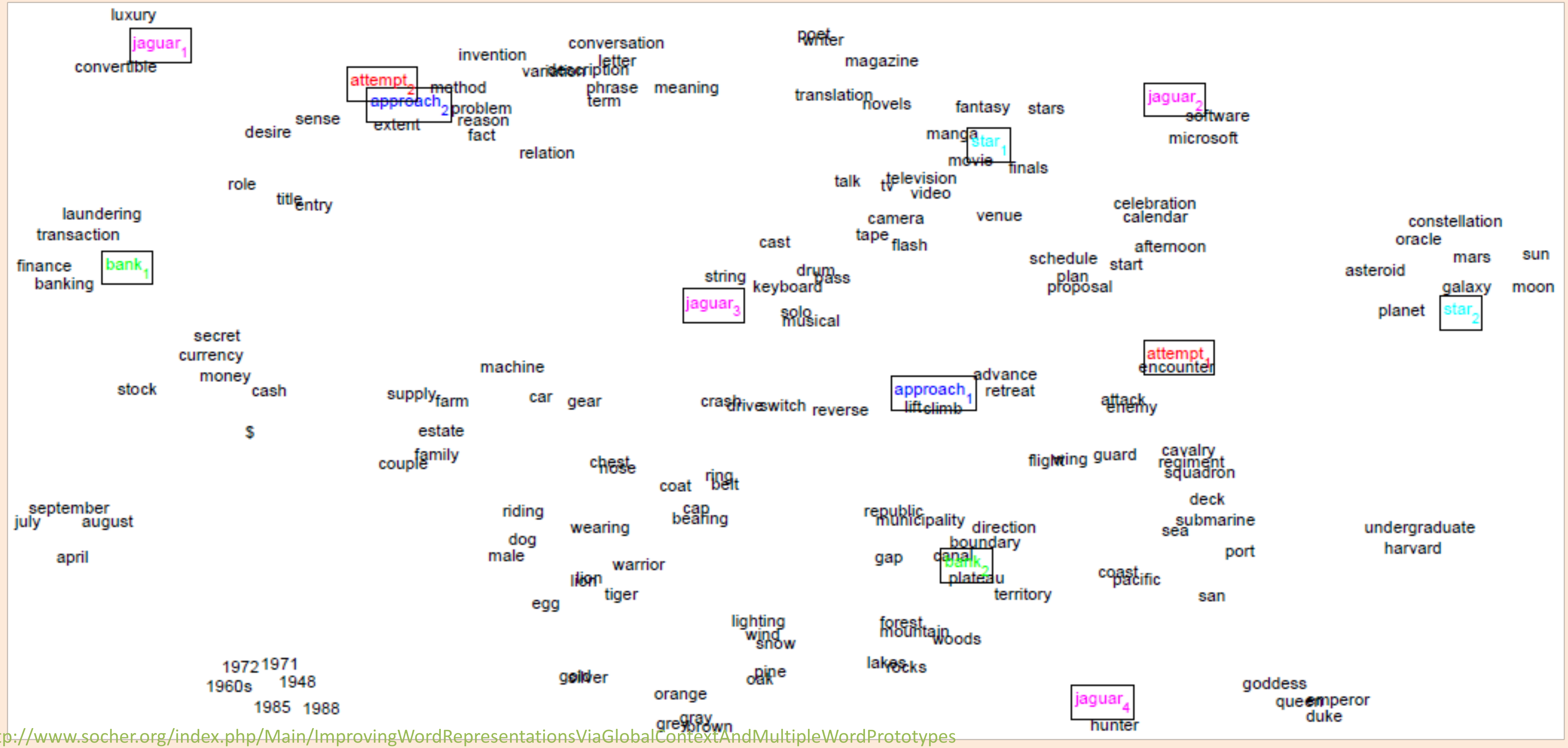
- What about homonyms and polysemy?
  - The word vectors would need to account for all meanings.

- More recent approaches:
  - Try to cluster the different contexts where words appear.
  - Use different vectors for different contexts.

$$X_{jaguar} = \begin{array}{c} \left[\begin{matrix} \cdots \\ \cdots \\ \vphantom{x} \end{matrix}\right] \end{array} \begin{matrix} z_{j1} \\ z_{j2} \\ z_{j3} \end{matrix}$$

bonus!

Multiple Word Prototypes



http://www.socher.org/index.php/Main/ImprovingWordRepresentationsViaGlobalContextAndMultipleWordPrototypes

bonus!

Why $z_i = Wx_i$?

- In PCA we had that the optimal $Z = XW^T(WW^T)^{-1}$.
- If W had normalized+orthogonal rows, $Z = XW^T$ (since $WW^T = I$).
  - So $z_i = Wx_i$ in this normalized+orthogonal case.

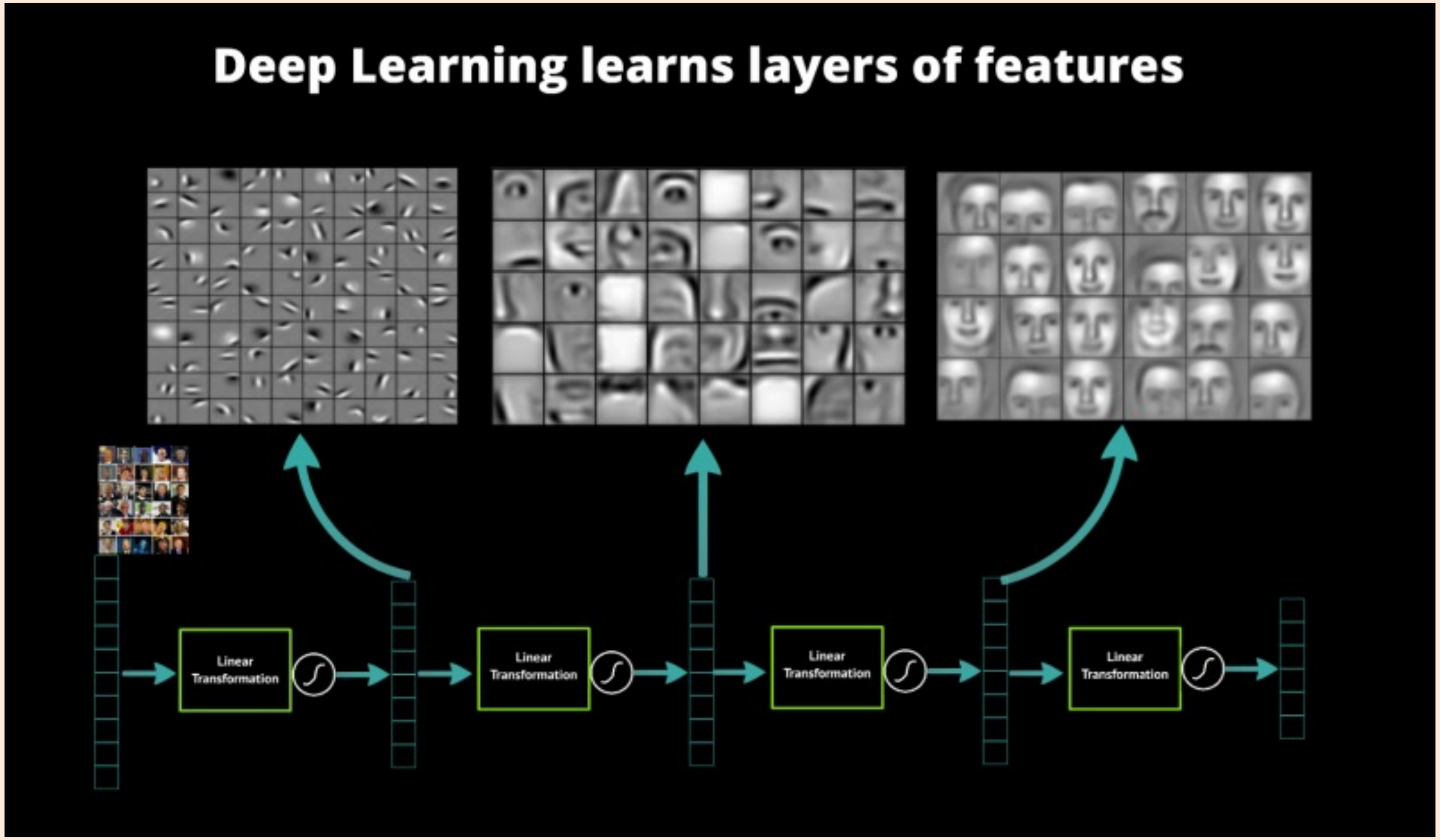
- Why we would use $z_i = Wx_i$ in neural networks?
  - We didn't enforce normalization or orthogonality.

- Well, the value $W^T(WW^T)^{-1}$ is just "some matrix".
  - You can think of neural networks as just directly learning this matrix.

bonus!

Cool Picture Motivation for Deep Learning

- Faces might be composed of different “parts”:

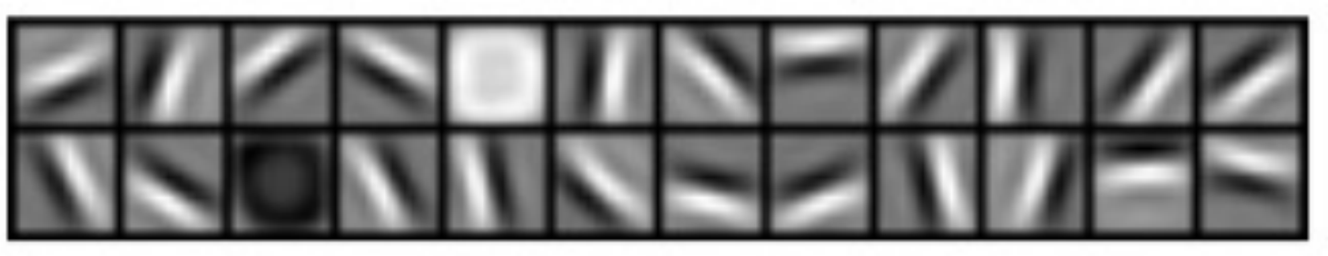


http://www.datarobot.com/blog/a-primer-on-deep-learning/

Cool Picture Motivation for Deep Learning

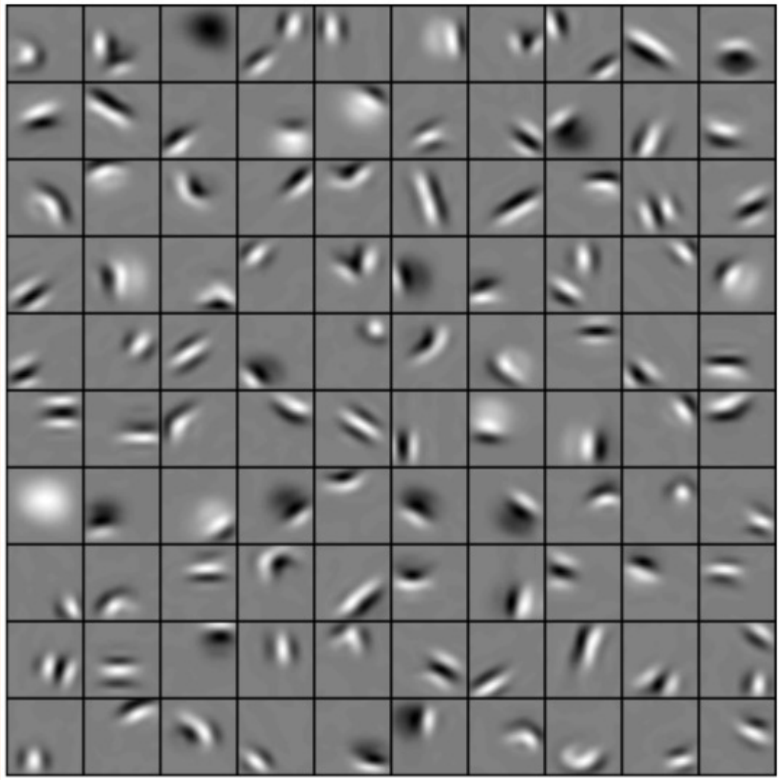
bonus!

- First layer of z_i trained on 10 by 10 image patches:



"Gabor filters"

- Attempt to visualize second layer:
  - Corners, angles, surface boundaries?



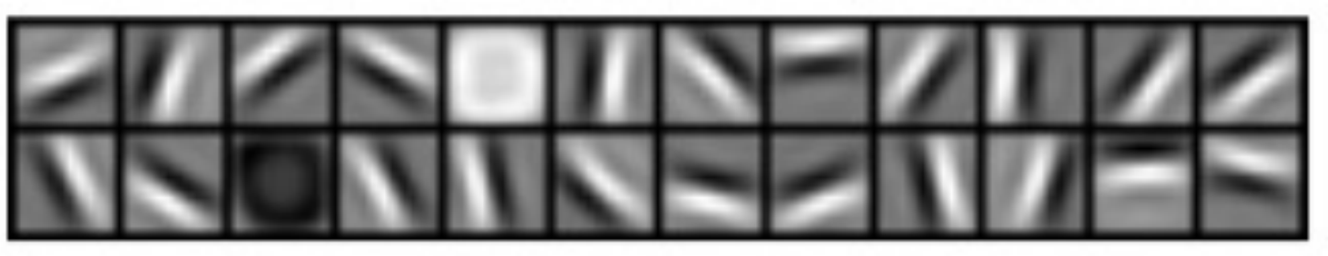
- Models require many tricks to work.
  - We'll discuss these next time.

http://www.cs.toronto.edu/~rgrosse/icml09-cdbn.pdf

Cool Picture Motivation for Deep Learning

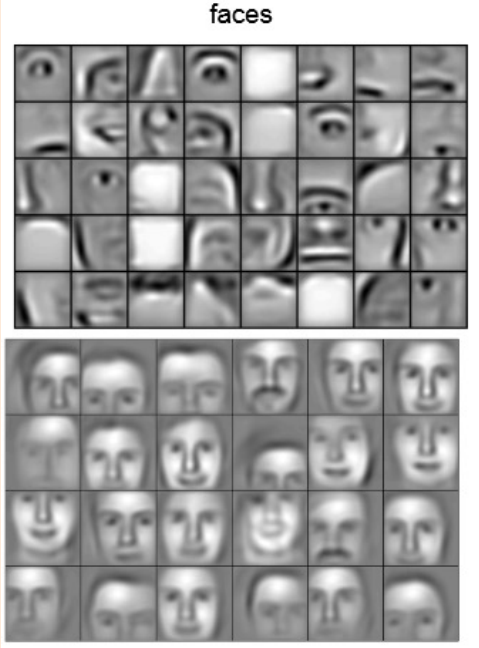
bonus!

- First layer of z_i trained on 10 by 10 image patches:



"Gabor filters"

- Visualization of second and third layers trained on specific objects:



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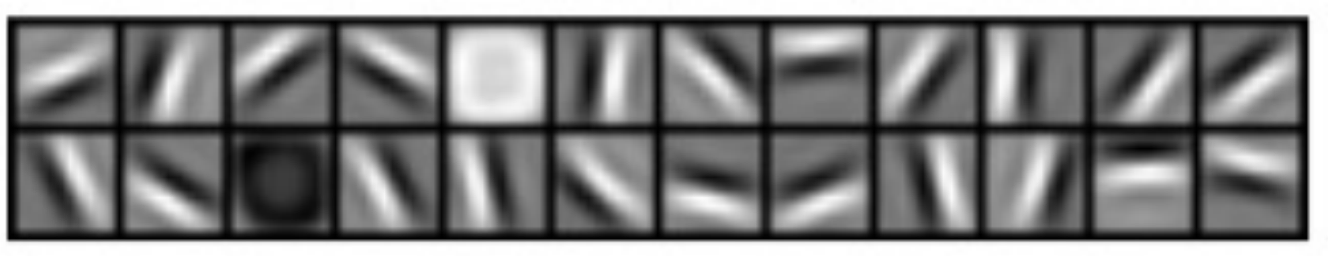

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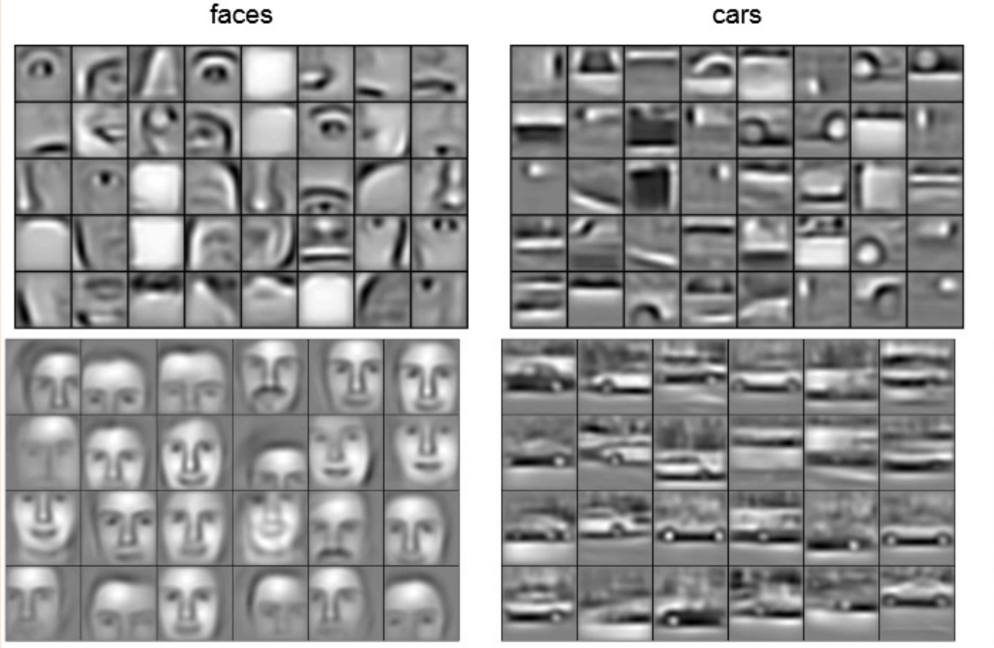

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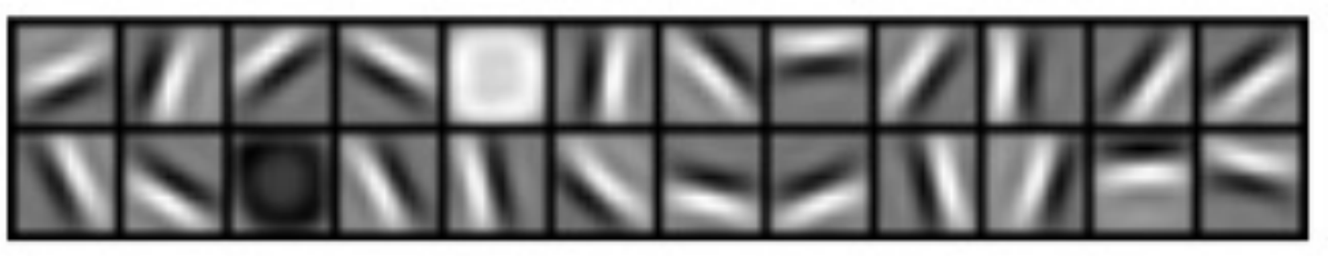

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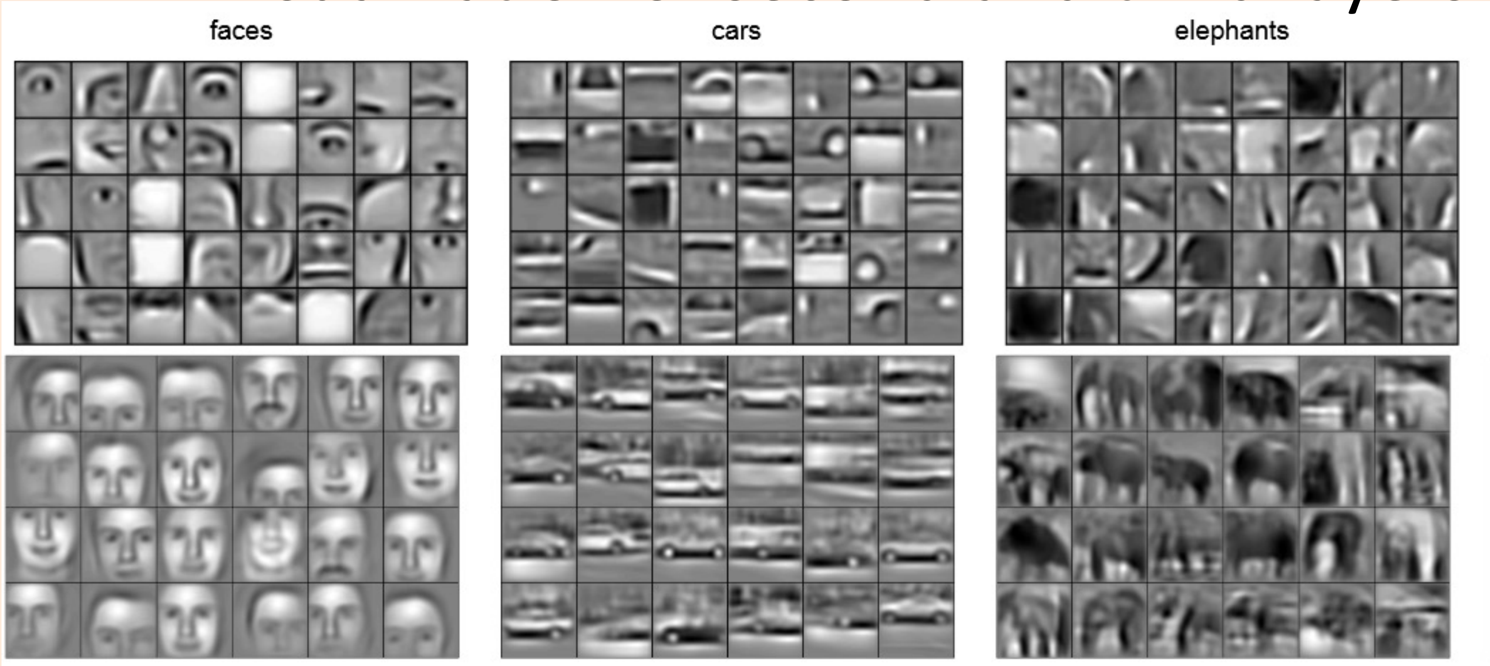

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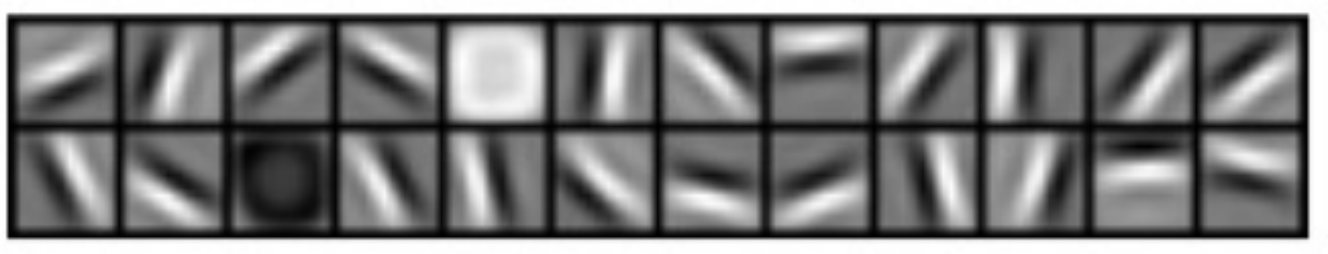

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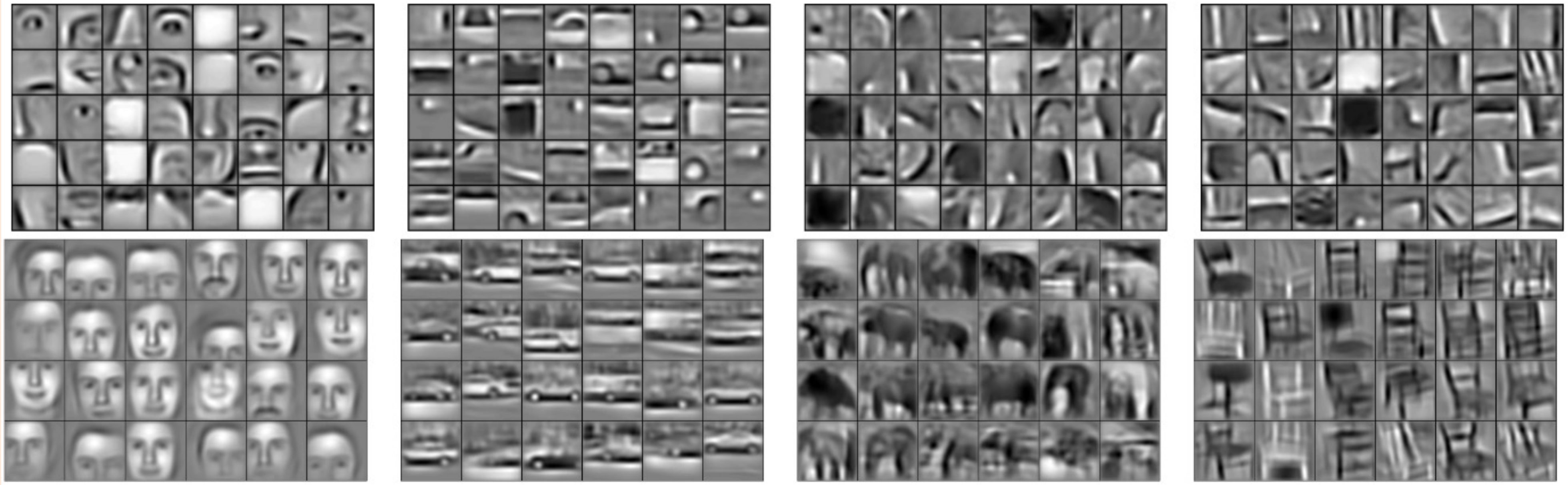

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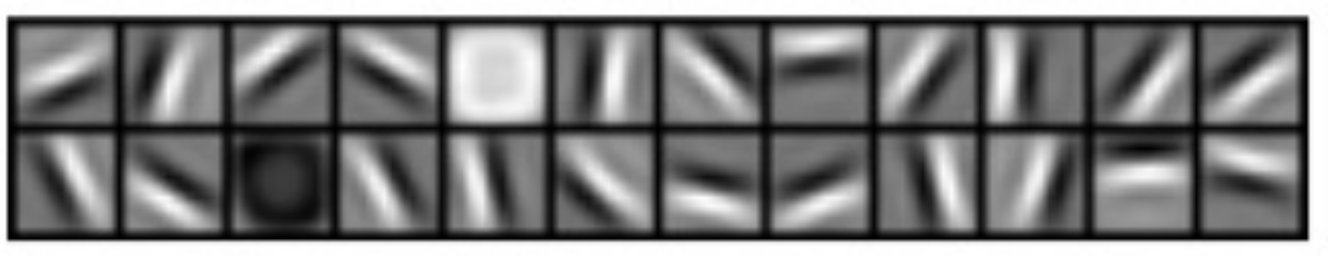

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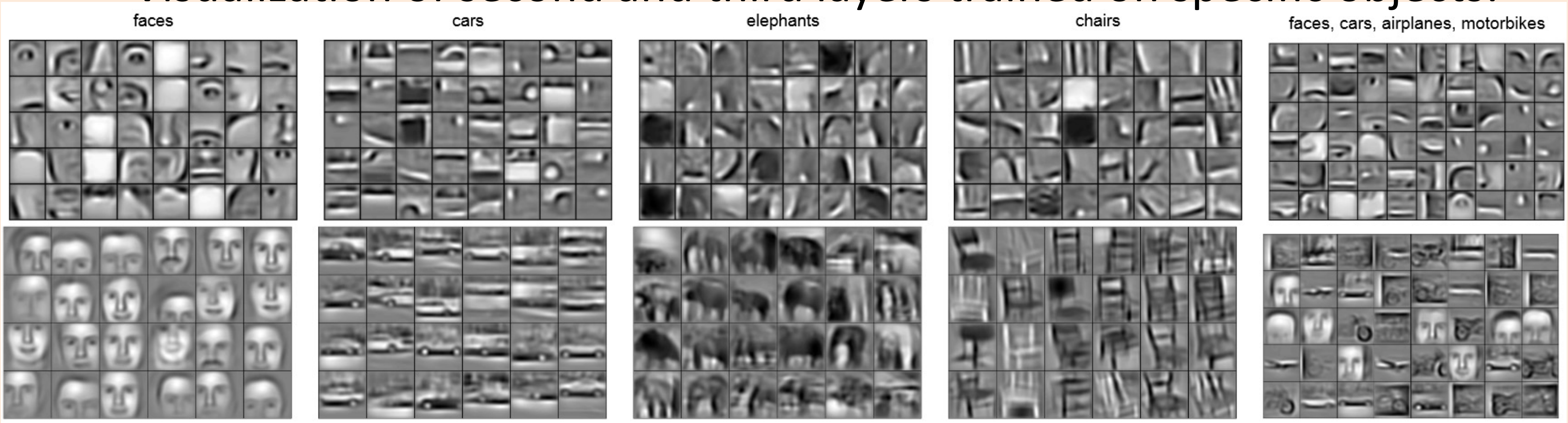

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